Volterra Gaussian processes, measure-preserving transformations and bridges.

A Volterra Gaussian process is a process of type $X_t = \int_0^t z_X(t,s)dW_s$, $t \ge 0$, where z_X is a deterministic kernel and W is a Brownian motion. We address the following 'linear problems' for X:

- 1. We derive measure-preserving transformations of X over [0,T]. These are maps \mathcal{T} such that $\text{Law}(\mathcal{T}(X)) = \text{Law}(X)$. Also, as an inherently, inversely related problem, we derive bridge transformations of X over [0,T], i.e. maps \mathcal{B} such that $\text{Law}(\mathcal{B}(X)) = \text{Law}(X|X_T = 0)$.
- 2. If X is self-similar, then we derive an alternative class of (ergodic) measure-preserving transformations of X over $[0, \infty)$.

In both cases, we investigate the information generated by the transformed processes. For X=W, both problems have been solved by Jeulin and Yor in 1990. For general X, we obtain the results by combining the known transformations for W and the special structure of X.