Stochastic Geometry and Wireless Network Modeling
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Introduction

The geometry of the location of nodes (mobile users, base stations, access points etc.) plays a key role in several classes of wireless communication networks, since it determines the signal to noise or signal to interference ratio for each potential communication and hence the possibility of establishing simultaneously some set of communications at a given bit rate.

Stochastic geometry provides a natural way of defining and computing macroscopic properties of such networks, by some averaging over all potential geometrical patterns for the nodes, in the same way as queuing theory provides averaged response times or congestion over all potential arrival patterns within a given parametric class.

The course will survey recent results obtained by this approach on two classes of wireless networks:

- mobile ad hoc networks (MANETs), where all nodes are essentially of the same type;
- cellular networks, where one distinguishes two or more types of nodes: concentration nodes and terminal nodes.

The aim of the course will be two fold:

1. To provide a concise introduction to relevant models of stochastic geometry:
   - Spatial shot noise processes;
   - Coverage processes;
   - Random tessellations.
2. To show how stochastic geometry allows one to analyze and optimize key features of these two classes of wireless networks:
   - Coverage and connectivity;
   - Power, admission and multiple access control;
   - Routing, diffusion or concentration of informations;
   - Capacity.

Course 1: Basic Stochastic Geometry Models

This course will review the definition and basic properties of Poisson point processes in the plane. We will review key operations on Poisson point processes (thinning, superposition, displacement) as well as key formulas like Campbell's formula.

We will show how to derive basic properties of a spatial shot noise process: its continuity properties, its Laplace transform, its moments. We will also analyze the law of max shot noise processes.

We will also focus on the Boolean model. Its basic coverage characteristics be reviewed. We will also give a brief account of its percolation properties.

Finally, we will review basic definitions and properties of Poisson Voronoi tessellations and cells. We will in particular discuss various random objects associated with bivariate point processes like for instance the set of points of the first point process that fall in a Voronoi cell w.r.t. the second point process.

Course 2: Signal to Interference Ratio Cells of a Poisson Point Process

Consider a marked point process of the Euclidean space, where the mark of a point is a positive random variable that represents its "transmission power".

Assume that the power radiated from a point decays in some isotropic way with Euclidean distance.

Define the signal to interference ratio (SIR) cell of a point to be the region of the space where the reception power from this point is larger than some increasing function of the interference. In this definition, the interference at some location of the space is just the sum of the reception powers from all other points.

This course will analyze a few basic stochastic geometry questions pertaining to such SIR cells in the case with independent marks:

- the volume and the shape of the typical cell;
- the properties of the coverage of the space by SIR cells, such as volume fraction;
- the law of the number of cells that cover a given location.
- the connections between these SIR cells and classical objects of stochastic geometry such as the Boolean model and Voronoi tessellations.

This course will be based on the paper :

"On a Coverage Process Ranging from the Boolean Model to the Poisson-Voronoi Tessellation", **Advances in Applied Probability**, 33, pp. 293-323, 2001, by the authors.

Course 3: Connectivity of MANETs

This course will study the impact of interferences on the connectivity of large-scale ad-hoc networks, using percolation theory. The set of nodes is represented by a Poisson point process of the plane. Assume that a connection can be set up between two nodes if the signal to noise and interference ratio at the receiver is larger than some threshold. The interference is the sum of the contributions of interferences from all other nodes weighted by a coefficient γ, which could be seen as the inverse of the processing gain in a CDMA (Code Division Multiple Access) context, and the noise is an external random field.

We will show that there is a critical value of γ above which the network is made of disconnected and finite clusters of nodes. We also prove that if γ

is non zero but small enough, there exist node spatial densities for which the network almost surely contains an infinite cluster of nodes, enabling distant nodes to communicate in multiple hops.

The shape of the region where such an infinite cluster exists is a function of the intensity of nodes and the γ parameter. It will in particular be shown that increasing the density of nodes may disconnect such a network, namely drive the network from the infinite cluster case to the disconnected finite cluster case.

This course will be based on the paper:

"Impact of Interferences on the Connectivity of Ad Hoc Networks", **IEEE Trans. Networking**, volume 13, number 2, pp 425–436, co-authored by O. Dousse, F.B. and P. Thiran.

Course 4: Power Control in Cellular Networks

Cellular networks involve a bivariate point process for representing the location of concentration nodes (e.g. base stations) and that of terminal nodes (users). The terminal node point processes is often assumed to be Poisson whereas the concentration node point process is either Poisson or periodic. In the simplest models, the terminal nodes associated with a given concentration node are those located in its Voronoi cell w.r.t. the point process of concentration nodes.

This course will focus on the case where terminal nodes require a fixed bit rate, and where power is controlled so as to maximize the number of terminal nodes that can be served by such a cellular network. In this case, powers become functionals of the underlying point processes.

We will first show how to estimate the number of terminal nodes with a predefined bit rate that have to be rejected from a static Poisson configuration because of power control infeasibility.

We will then study a pure-jump Markov generator which can be seen as a generalization of the spatial birth-and-death generator and which allows to represent the arrival, mobility and departure of terminal nodes, and which can be used to model the dynamics of such power controlled cellular wireless communication networks. From the analysis of this generator, we will deduce an expression for the blocking probability in such wireless networks. This expression can be seen as a spatial version of the classical Erlang loss formula.

We will also discuss the case of elastic traffic (ongoing work).

This course will be based on the following papers:

“Spatial Averages of Coverage Characteristics in Large CDMA Networks”, **ACM WINET (Wireless Networks)**, 8, 2002. by F.B., B.B. and F. Tournois,

“Up and Downlink Admission and Congestion Control and Maximal Load in Large Homogeneous CDMA Networks” **ACM MONET**, Vol. 9, No. 6, Dec. 2004, by F.B., B.B. and M. Karray,

“Blocking Rates in Large CDMA Networks via a Spatial Erlang Formula”, **Proceedings of IEEE Infocom’05**, 2005, Miami, co-authored by F.B., B.B. and M. Karray.

Course 5: Multiple Access Control in MANETs

In this course, we will analyze an Aloha type access control mechanism for large MANETs. The access scheme is designed for the multihop context, where it is important to find a compromise between the spatial density of communications and the range of each transmission. More precisely, the analysis aims at optimizing the product of the number of simultaneously successful transmissions per unit of space (spatial reuse) by the average range of each transmission. The optimization is obtained via an averaging over all Poisson configurations for the location of interfering nodes, where an exact evaluation of signal over noise ratio is possible. The main mathematical tools are spatial versions of the so-called additive and max shot noise processes. The resulting MAC protocol exhibits some interesting properties. In particular, its transport capacity is proportional to the square root of the density of nodes.

This course will be based on the paper:

“An Aloha Protocol for Multihop Mobile Wireless Networks”, **IEEE Transactions on Information Theory**, Vol. 52, No. 2, pp. 421-436, 2006, co-authored by F.B., B.B. and P. Muhlethaler.

Course 6: Routing

In this course, we will analyze a class of spatial random spanning trees built on a realization of an homogeneous Poisson point process of the plane. This tree has a simple radial structure with the origin as its root This class of spanning trees has applications in

- multihop diffusion from a given node in MANETs;
- multihop routing to a given node in MANETs;
- concentration in wireless sensor communication networks where information has to be gathered at a central node.

We will first show how to use stochastic geometry arguments to analyze local functionals of the random tree such as the distribution of the length of its edges or the mean degree of its nodes. Far away from the origin, these local properties are shown to be close to those of the directed spanning tree introduced by Bhatt and Roy.

We will then use the theory of continuous state space Markov chains to analyze some non local properties of the tree such as the shape and structure of its semi-infinite paths or routes, the shape of the set of its nodes less than k generations away from the origin etc.. We will also stress the differences that exist between several types of averages like path averages and space averages.

This course will be based on the paper:

"The Radial Spanning Tree of a Poisson Point Process", Proceedings of the 43^{th} Allerton Conference, 2005, Illinois University at Urbana Champaign, submitted to **Annals of Applied Probab.** co-authored by F.B. and C. Bordenave.