Abstract: Consider the infinite tree $T = \cup_{n\geq 0} N^n$ in which each edge connecting a node v with a node v_i is carrying a weight $T_i(v)$. The infinite vectors $T(v) = (T_1(v), T_2(v), ...)$ are assumed to be i.i.d. Given a path from the root $\emptyset$ to a node $v = v_1...v_n$ let

$$L(v) \;=\; T_{v_1}(\emptyset)T_{v_2}(v_1) \cdot ... \cdot T_{v_n}(v_1 \ldots .v_{n-1})$$

be the weight accumulated along this path. The associated weighted branching process $(Z_n)_{n\geq 0}$ is then defined by $Z_0 = 1$ and

$$Z_n \;=\; \sum_{|v|=n} L(v),$$

for $n \geq 1$, where $|v|$ means the length of v. The most classical example of such a process is the Galton-Watson branching process where all weights are 0 or 1. But there are other interesting examples, for instance the size of a random Cantor set obtained by a so-called random recursive construction or the asymptotic size of sublists obtained by an application of the Quicksort sorting algorithms.

Weighted branching processes are also related to the following stochastic fixed point equation:

$$\mathcal{L}(W) \;=\; \mathcal{L}\left(\sum_{i\geq 1} T_i W_i\right)$$

where $T = (T_1, T_2, ...)$ denotes a generic copy of the $T(v)$, $W, W_1, ...$ are i.i.d. and independent of T, and where $(\cdot)$ means the distribution of a random variable. In fact, if all T_i are nonnegative and $\sum_{i\geq 1} ET_i = 1$ then $(Z_n)_{n\geq 0}$ is a nonnegative martingale with a.s. limit W being a (possibly trivial) fixed point of the above equation.

The lecture series will give an introduction of a number of interesting aspects of the weighted branching model and the afore-mentioned stochastic fixed point equation. Problems addressed are the convergence of the weighted branching process, related martingale problems, existence and uniqueness of nontrivial fixed points and also certain renewal theoretic connections. Own contributions which are presented in the course have been developped in collaboration with Uwe Rösler (Kiel) and my Ph.D. student Dirk Kuhlbusch (Münster).