

similar mathematical model. Three problems in this field which are at present under investigation are: studies of the passage of space vehicles through the ionosphere; studies of the stream of high speed particles (known as the solar wind) ejected by the Sun and passing near the Earth; studies of the tenuous airstream past a body moving at very high speeds. In all these examples, the theoretical approach is of paramount importance to our hopes of increasing understanding.

#### **Experience and language in partnership**

The history of man is in many ways a history of technology. As each new technical development was exploited so man gained stature among his fellow creatures and power over his environment. Whilst one would not wish to suggest that our primeval ancestors used mathematical models to develop the stone axe, yet it is fair to claim for mathematics a very considerable share of the credit for the developing space programmes of the major nations. Clearly engineering and mathematics are now in partnership and the interplay between them can perhaps be best described in the following brief quotation from an unpublished paper of Richard von Mises, who until his death in 1953 was Gordon McKay Professor of Aerodynamics and Applied Mathematics at Harvard University. He wrote, "The leitmotif, the ever-recurring melody, is that two things are indispensable in any reasoning, in any description we shape of a segment of reality; to submit to experience and to face the language that is used, with unceasing logical criticism."

## **The Algebra of Genealogy**

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Recently Mr E. A. Leeb wrote as follows to my colleague Professor J. W. S. Cassels. "There is an old conundrum, 'Brothers and sisters have I none, yet this man's father is my father's son'. While it is possible to solve the problem by trial-and-error, I was wondering whether the relationships involved could be algebraically expressed. Or, to consider the broader question, can familial relationships be written mathematically?"

Professor Cassels showed the letter to me, and my approach to the problem is printed overleaf. Before turning the page, the reader may care to explore Mr Leeb's two questions for himself.

Let us denote individuals by  $u, v, \dots, u = v$  denoting identity. Let us define a mapping  $F$  from the set of all individuals into itself, by writing  $Fx$  for 'the father of  $x$ '. This is well defined (if we are evolutionists rather than fundamentalists), although  $Fx$  may, of course, not be *known* to be so describable. The equation  $Fx = Fy$  asserts that  $x$  and  $y$  have the same father, and this situation can arise in *just three ways*:

- (i)  $x$  and  $y$  may be identical;
- (ii)  $x$  and  $y$  may be full sibs (i.e., have the same father *and* mother);
- (iii)  $x$  and  $y$  may have the same father but different mothers.

To say that  $u$  is the son of  $v$  can be written, in our notation, as  $Fu = v$ , provided that the data tell us that  $v$  is male.

Now let  $w$  be the speaker, and  $z$  the person spoken of; we do not know at the outset that  $w$  and  $z$  are distinct. We are told that  $z$  is male, and we are told that his father,  $Fz$ , is 'my father's son'; i.e., that

$$F^2 z = Fw.$$

Thus  $x = Fz$  and  $y = w$  satisfy  $Fx = Fy$ , and we have already analysed that equation exhaustively. Case (i) gives us the traditional solution to the problem:  $x = y$ , so  $Fz = w$ , so ' $F$  am male and *'this man'* is my son. Case (ii) is excluded by the first line of the rhyme, for if  $x$  and  $y$  (that is,  $Fz$  and  $w$ ) were full sibs, then ' $F$ ' would have a brother. We are left with case (iii), which on its simplest interpretation yields the following further and usually overlooked solution:

*'F may be male or female; my father had at least two wives (legally or not); I am his child by one wife and 'this man' is his grandson by another wife.*

(The situation may be more complicated than this, because of the multiple relationships associated with more or less extreme forms of inbreeding such as the brother-sister marriages traditional in some of the royal families of the ancient world.)

I was surprised to get this second solution, for I have known the conundrum for some forty years and always believed the customary solution to be unique.

Mr Leeb's closing query is particularly timely, because genealogists are currently much concerned about the problem of reconstructing 'family trees' on a computer. Work of this sort is being done, for example, by the population geneticists in Pavia and in Cambridge (reference 1), by the historical demographers in Cambridge (reference 2), and by the genealogists in Salt Lake City. (See also Newcombe (reference 3).) It is clear that a trial or final 'tree' cannot be held in the computer, much less retrieved from it, unless its nodes are adequately labelled, and the non-trivial task of devising an efficient and readily comprehensible labelling, and writing machine programs in terms thereof, is very close to that proposed by Mr. Leeb.

## References

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## Gambling and Probability: Some Early Problems<sup>1</sup>

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Students of chance, faced with a difficult problem in probability theory, may find it amusing to recall that the foundations of the subject, laid in the 16th and 17th centuries, were based on the frivolities of gambling. Today, gambling still provides minor motivation for the study of probability theory. Not long ago, Dr E. O. Thorp, an American mathematician, after studying the game of 'twenty one' was able to beat the banks in Las Vegas casinos, and accumulate a small fortune!

The origins of gambling are lost in ancient history. Early Egyptian tomb paintings depict board games played with sheep's heel-bones, known as astragals, which could fall on one of four faces, and served as imperfect dice. These astragals gradually evolved into regular six-sided cubic dice some time before the Christian era; such primitive dice have been found in many archaeological excavations, including some in England dating from the Roman invasion. Both astragals and dice survived into the Middle Ages, when dicing games with rules very similar to those played today became firmly established.

The calculus of probabilities owes much to early gamblers; they were able to record from their very wide experience that certain throws of (one, two or more) astragals and dice were more likely to occur than others. Their practical knowledge greatly assisted the mathematicians who were attempting to formulate and solve the early problems of probability theory.

One of the earliest of these was Gerolamo Cardano (1501-1576), whose name is perhaps best known for his dispute with Tartaglia concerning the solution of

<sup>1</sup> This article formed part of a talk at the *Mathematics of Today* Conference for sixth formers, organised by the Department of Mathematics, University of Southampton, 14-16 April 1971.