



Let $N_1(t)$ denote the no. of events of type I in $(0, t]$ and $N_2(t)$ the no. of type II events.

$$\text{Then } N(t) = N_1(t) + N_2(t)$$

Prop. 5.2. $N_1(t), t \geq 0$ and $N_2(t), t \geq 0$ are both Poisson processes with rates λp and $\lambda(1-p)$, respectively. Furthermore, they are independent.
 N_1, N_2 are thinned versions of N .

Pf. Check def. 5.3. for N_1 :

- $N_1(0) = 0$ since $N(0) = 0$.

- $P(N_1(h) = 1) = P(N_1(h) = 1 \mid N(h) = 1)$.

- $P(N(h) = 1) = p \cdot (\lambda h + \sigma(h)) = \lambda p h + \sigma(h)$

$$P(N_1(h) \geq 2) \leq P(N(h) \geq 2) = \sigma(h)$$

- $N_1(t)$ has stationary and independent