Or, if W is your waiting time and S is your service time:

$$T = W + S$$

$$E(T) = E(W) + E(S)$$

$$= \frac{1}{\lambda_1 + \lambda_2} + E(S \mid R_1 \le R_2) \cdot \mathbb{P}(R_1 \le R_2)$$

mean of $\min(R_1, R_2)$

$$E(S \mid R_1 > R_2) = \frac{1}{\lambda_1 + \lambda_2} + \frac{1}{\lambda_1} \cdot \frac{\lambda_1}{\lambda_1 + \lambda_2} + \frac{1}{\lambda_2} \cdot \frac{\lambda_2}{\lambda_1 + \lambda_2}$$

$P(R_1 > R_2)$

$$= \frac{3}{\lambda_1 + \lambda_2}.$$

5.3. The Poisson Process

5.3.1. A stochastic process $N(t)$, $t \ge 0$, is a <u>counting process</u> if $N(t)$ represents the total no. of events occurring by time t.

Ex. (a) No. of customers entering a store by time t.

(b) "Event" = time of childbirth
$N(t)$ = no. of children born in $(0, t]$

(c) $N(t)$ = no. of goals a certain football player has scored by time t

By def. a counting process

1. $N(t) \geq 0$
2. $N(t)$ is integer-valued
3. If $s < t$ then $N(s) \leq N(t)$
4. For $s < t$, $N(t) - N(s)$ is the number of events in $(s, t]$.

$N(t)$ has <u>independent increments</u> if the no. of events in disjoint intervals are independent random variables.

$N(t)$ has <u>stationary increments</u> if the no. of events in an interval depends only on its length. (I.e., the prob. distribution depends only on its length:

$$N(100+t) - N(100)$$
$$N(1+t) - N(1)$$

has the same distribution.

Def 5.2. The function f is said to be $o(h)$ "little o of h" if

$$\lim_{h \to 0} \frac{f(h)}{h} = 0$$

"f goes faster to 0 than h"

Ex. x^2 is $o(h)$

$x\sqrt{x}$ is $o(h)$

$5x$ is not $o(h)$

$7x\sqrt{x}$ is $o(h)$

If f is $o(h)$ and g is $o(h)$ then so is $f+g$:

$$\frac{(f+g)(h)}{h} = \frac{f(h)}{h} + \frac{g(h)}{h}$$

Also $c \cdot f$ is $o(h)$:

$$\frac{(cf)(h)}{h} = c \cdot \frac{f(h)}{h}$$