Simulation principle

Let U be uniform on $[0,1]$. If X is a continuous random variable with strictly increasing distribution function F, then $F(X)$ is uniform. Thus $F^{-1}(U)$ has distribution F.

Proof: $\mathbb{P}(F^{-1}(U) \le x) = \mathbb{P}(U \le F(x))$

$= F(x)$ if $0 \le F(x) \le 1$.

$\mathbb{P}(F(X) \le u) = \mathbb{P}(X \le F^{-1}(u)) = F(F^{-1}(u))$

$= u$ for $0 \le u \le 1$.