

$$3. a) U = \{ (x_1, x_2, x_3, x_4)^T : x_1 + 2x_2 + 3x_3 + x_4 = 0 \} = N((1 \ 2 \ 3 \ 1))$$

$$\left( \begin{array}{ccc|c} \textcircled{1} & 2 & 3 & 1 \\ & s & t & u \end{array} \right) \begin{cases} x_1 = -2s - 3t - u \\ x_2 = s \\ x_3 = t \\ x_4 = u \end{cases}, \quad x = s \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ 0 \\ 1 \\ 0 \end{pmatrix} + u \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Bas i U:  $\{u_1, u_2, u_3\}$ .

$$b) V = N\left(\begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & 2 & 1 & 1 \end{pmatrix}\right)$$

$$\left( \begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 2 & 1 & 1 \end{array} \right) \xrightarrow{\text{Bo}^3} \left( \begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & \textcircled{1} & 1/2 & 1/2 \end{array} \right) \xrightarrow{\text{Bo}^1} \left( \begin{array}{ccc|c} 1 & 0 & 5/2 & -1/2 \\ 0 & 1 & 1/2 & 1/2 \end{array} \right) \begin{cases} x_1 = -\frac{5}{2}s + \frac{1}{2}t \\ x_2 = -\frac{1}{2}s - \frac{1}{2}t \\ x_3 = s \\ x_4 = t \end{cases}$$

$$\therefore x = \frac{s}{2} \begin{pmatrix} -5 \\ -1 \\ 2 \\ 0 \end{pmatrix} + \frac{t}{2} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 2 \end{pmatrix} \quad \text{Bas i V: } \{v_1, v_2\}$$

$$c) U \cap V = N\left(\begin{pmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 3 & 0 \\ 0 & 2 & 1 & 1 \end{pmatrix}\right)$$

$$\left( \begin{array}{ccc|c} \textcircled{1} & 2 & 3 & 1 \\ 1 & 1 & 3 & 0 \\ 0 & 2 & 1 & 1 \end{array} \right) \xrightarrow{\text{Bo}^1} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & \textcircled{-1} & 0 & -1 \\ 0 & 2 & 1 & 1 \end{array} \right) \xrightarrow{\text{Bo}^3} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & \textcircled{1} & -1 \end{array} \right)$$

$$\xrightarrow{\text{Bo}^1} \left( \begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & \textcircled{1} & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array} \right) \xrightarrow{\text{Bo}^1} \left( \begin{array}{ccc|c} \textcircled{1} & 0 & 0 & 2 \\ 0 & \textcircled{1} & 0 & 1 \\ 0 & 0 & \textcircled{1} & -1 \end{array} \right) \begin{matrix} F \\ S \end{matrix}$$

$$\begin{cases} x_1 = -2s \\ x_2 = -s \\ x_3 = s \\ x_4 = s \end{cases}$$

Bas i U ∩ V:  $\{(-2 \ -1 \ 1 \ 1)^T\}$