(22.5.07)

Ex Bestäm projektionsmatrisen på planet

$$2x_1 - 3x_2 + x_3 = 0 \quad i \quad \mathbb{R}^3.$$

Räkna också ut speglingsmatrisen i detta plan.

U = planet $2x_1 - 3x_2 + x_3 = 0$. Sätt $a = (2\ -3\ 1)^T$.

Då är $U^\perp = \operatorname{spn}\{a\}$.

Projektionsmatrisen på $U^\perp$ är:

$$\underline{P'} = \frac{aa^T}{\|a\|^2} = \frac{1}{14}\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}(2\ -3\ 1) = \underline{\frac{1}{14}\begin{pmatrix} 4 & -6 & 2 \\ -6 & 9 & -3 \\ 2 & -3 & 1 \end{pmatrix}}.$$

Projektionsmatrisen på $\overline{U}$ blir då:

$$\underline{\underline{P}} = I - P' = \frac{1}{14}\begin{pmatrix} 14 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 14 \end{pmatrix} - \frac{1}{14}\begin{pmatrix} 4 & -6 & 2 \\ -6 & 9 & -3 \\ 2 & -3 & 1 \end{pmatrix}$$

$$= \underline{\underline{\frac{1}{14}\begin{pmatrix} 10 & 6 & -2 \\ 6 & 5 & 3 \\ -2 & 3 & 13 \end{pmatrix}}}.$$

Speglingsmatrisen i $\overline{U}$:

$$\underline{\underline{S}} = 2P - I = \frac{1}{7}\begin{pmatrix} 10 & 6 & -2 \\ 6 & 5 & 3 \\ -2 & 3 & 13 \end{pmatrix} - \frac{1}{7}\begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

$$= \underline{\underline{\frac{1}{7}\begin{pmatrix} 3 & 6 & -2 \\ 6 & -2 & 3 \\ -2 & 3 & 6 \end{pmatrix}}}. \qquad (= I - 2P')$$

Ex) (22.5.07)

```
In[1]:= P = {{10, 6, -2}, {6, 5, 3}, {-2, 3, 13}} / 14
```

Out[1]= $\left\{\left\{\frac{5}{7}, \frac{3}{7}, -\frac{1}{7}\right\}, \left\{\frac{3}{7}, \frac{5}{14}, \frac{3}{14}\right\}, \left\{-\frac{1}{7}, \frac{3}{14}, \frac{13}{14}\right\}\right\}$

```
In[2]:= S = 2 P - IdentityMatrix[3]
```

Out[2]= $\left\{\left\{\frac{3}{7}, \frac{6}{7}, -\frac{2}{7}\right\}, \left\{\frac{6}{7}, -\frac{2}{7}, \frac{3}{7}\right\}, \left\{-\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right\}\right\}$

```
In[34]:= plane = Line[{{0, 0, 0}, {1, 1, 1}, {-3 / 2, 0, 3}, {0, 0, 0}}]
```

Out[34]= $\mathrm{Line}\left[\left\{\{0, 0, 0\}, \{1, 1, 1\}, \left\{-\frac{3}{2}, 0, 3\right\}, \{0, 0, 0\}\right\}\right]$

```
In[11]:= x = {-1 / 3, 2 / 3, 1}
```

Out[11]= $\left\{-\frac{1}{3}, \frac{2}{3}, 1\right\}$

```
In[14]:= xp = P.x
```

Out[14]= $\left\{-\frac{2}{21}, \frac{13}{42}, \frac{47}{42}\right\}$

```
In[15]:= xs = S.x
```

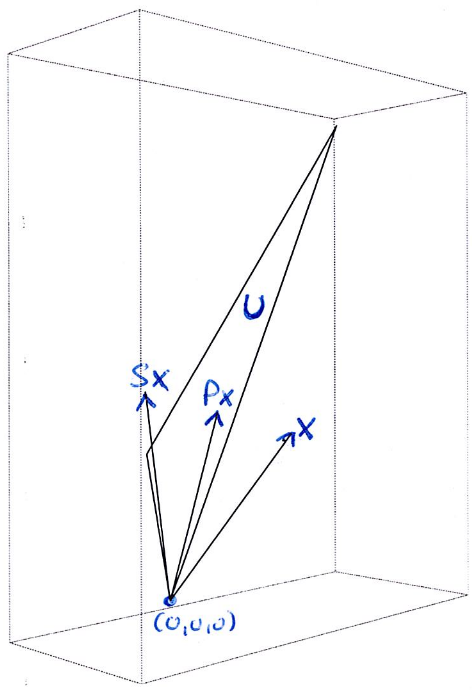
Out[15]= $\left\{\frac{1}{7}, -\frac{1}{21}, \frac{26}{21}\right\}$

```
In[18]:= linex = Line[{{0, 0, 0}, x}]; linexp = Line[{{0, 0, 0}, xp}]; linexs = Line[{{0, 0, 0}, xs}]
```

Out[18]= $\mathrm{Line}\left[\left\{\{0, 0, 0\}, \left\{\frac{1}{7}, -\frac{1}{21}, \frac{26}{21}\right\}\right\}\right]$

```
In[44]:= Show[Graphics3D[{plane, linex, linexp, linexs}], ViewPoint → {1.5, 1.5, 0}]
```

Out[44]=



Ex | (20.5.08) Ortonormera vektorerna

$$a_1 = (2\ 2\ 0)^T,\quad a_2 = (4\ 2\ 1)^T,\quad a_3 = (2\ 0\ 4)^T$$

med hjälp av Gram-Schmidt-proceduren till ett ortonormelt system.

Steg 1: Sätt: $\underline{v_1} = a_1 = \underline{\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}}$.

Sätt: $v_2 = a_2 - \lambda \cdot v_1$

Krav: $0 = v_1^T v_2 = \underbrace{v_1^T a_2}_{=12} - \lambda \underbrace{\|v_1\|^2}_{8}$. $\therefore \underline{\underline{\lambda = \frac{3}{2}}}$.

$$\therefore \underline{v_2} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} - \frac{3}{2}\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} = \underline{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}}.$$

Sätt: $v_3 = a_3 - \lambda_1 v_1 - \lambda_2 v_2$. Kräver att:

$$\begin{cases} 0 = v_1^T v_3 = \underbrace{v_1^T a_3}_{=4} - \lambda_1 \underbrace{\|v_1\|^2}_{8} \\ 0 = v_2^T v_3 = \underbrace{v_2^T a_3}_{6} - \lambda_2 \underbrace{\|v_2\|^2}_{3} \end{cases}, \quad \therefore \underline{\underline{\begin{cases} \lambda_1 = \frac{1}{2} \\ \lambda_2 = 2 \end{cases}}}$$

$$\therefore \underline{v_3} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \underline{\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}}.$$

Steg 2: Sätter:

$$\begin{cases} q_1 = \dfrac{v_1}{\|v_1\|} = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \\ q_2 = \dfrac{v_2}{\|v_2\|} = \dfrac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \\ q_3 = \dfrac{v_3}{\|v_3\|} = \dfrac{1}{\sqrt{6}}\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \end{cases}$$

$\therefore$ $\underline{\{q_1, q_2, q_3\} \text{ ett ON-system}}$.

$\boxed{\text{Ex}}$ $A = \begin{pmatrix} 2 & 4 & 2 \\ 2 & 2 & 0 \\ 0 & 1 & 4 \end{pmatrix}$ (jmfr. föregående exempel)

$\qquad\quad a_1 \quad a_2 \quad a_3$

$$\begin{cases} a_1 = v_1 & = 2\sqrt{2}\cdot q_1 \\ a_2 = \frac{3}{2}v_1 + v_2 & = 3\sqrt{2}\cdot q_1 + \sqrt{3}\cdot q_2 \\ a_3 = \frac{1}{2}v_1 + 2v_2 + v_3 & = \sqrt{2}\cdot q_1 + 2\sqrt{3}\,q_2 + \sqrt{6}\cdot q_3 \end{cases}$$

<u>Sätter</u>: $A = (a_1 \ a_2 \ a_3)$, $Q = (q_1 \ q_2 \ q_3)$.

<u>Då gäller</u>:

$$A = QR = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{3} & -1/\sqrt{6} \\ 1/\sqrt{2} & -1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & 2/\sqrt{6} \end{pmatrix} \begin{pmatrix} 2\sqrt{2} & 3\sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} & 2\sqrt{3} \\ 0 & 0 & \sqrt{6} \end{pmatrix}$$

<u>QR-faktoriseringen av A</u>.