Ex $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1\cdot 4 - 2\cdot 3 = 4-6 = \underline{-2}$.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1\cdot 5\cdot 9 + 2\cdot 6\cdot 7 + 3\cdot 4\cdot 8 - 3\cdot 5\cdot 7 - 1\cdot 6\cdot 8 - 2\cdot 4\cdot 9 \underline{= 0}.$$

Sarrus regel:

$$\left|\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{matrix}\right| \begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix}$$

Ex $\begin{vmatrix} 2 & -1 & 4 \\ 1 & 0 & 2 \\ -6 & 3 & -12 \end{vmatrix} \overset{4.}{=} (-3)\cdot \begin{vmatrix} 2 & -1 & 4 \\ 1 & 0 & 2 \\ 2 & -1 & 4 \end{vmatrix} \overset{3.}{=} 0$

$\left(\text{eller} \quad \overset{4.}{=} 2\cdot \begin{vmatrix} 2 & -1 & 2 \\ 1 & 0 & 1 \\ -6 & 3 & -6 \end{vmatrix} \overset{3.}{=} 0 \right)$

Ex $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 7 & 8 & 0 \end{vmatrix} \overset{5.}{=} \begin{vmatrix} 1 & 2 & 3+3 \\ 4 & 5 & 6+4 \\ 7 & 8 & 9+0 \end{vmatrix}$

$= \begin{vmatrix} 1 & 2 & 6 \\ 4 & 5 & 10 \\ 7 & 8 & 9 \end{vmatrix} = 15$, (Sarrus regel, kolla !!)

Ex

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad A^T = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$(b_{ij} = a_{ji})$

A		A^T	
Elementär prod.	**$\varepsilon(\sigma)$**	**Elementär prod.**	**$\varepsilon(\sigma)$**
$a_{11}a_{22}a_{33}$	$+1$	$\underbrace{b_{11}b_{22}b_{33}}_{= a_{11}a_{22}a_{33}}$	$+1$
$a_{11}a_{23}a_{32}$	-1	$\underbrace{b_{11}b_{23}b_{32}}_{= a_{11}a_{32}a_{23}}$	-1
$a_{12}a_{21}a_{33}$	-1	$\underbrace{b_{12}b_{21}b_{33}}_{= a_{21}a_{12}a_{33}}$	-1
$a_{12}a_{23}a_{31}$	$+1$	$\underbrace{b_{12}b_{23}b_{31}}_{= a_{21}a_{32}a_{13}}$	$+1$
$a_{13}a_{21}a_{32}$	$+1$	$\underbrace{b_{13}b_{21}b_{32}}_{= a_{31}a_{12}a_{23}}$	$+1$
$a_{13}a_{22}a_{31}$	-1	$\underbrace{b_{13}b_{22}b_{31}}_{= a_{31}a_{22}a_{13}}$	-1

$$\therefore \det(A) = \det(A^T).$$

Ex

$$A = \begin{array}{ccc} a_1 & a_2 & a_3 \\ \end{array}$$
$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad A' = \begin{pmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{pmatrix}$$
$$\begin{array}{ccc} a_1 & a_3 & a_2 \end{array}$$

(byt plats på a_2 och a_3)

A: Elementär prod.	$\varepsilon(\sigma)$	A': Elementär prod.	$\varepsilon(\sigma)$
$a_{11}a_{22}a_{33}$	$+1$	$\underbrace{a'_{11}a'_{22}a'_{33}}_{} = a_{11}a_{23}a_{32}$	$+1$
$a_{11}a_{23}a_{32}$	-1	$\underbrace{a'_{11}a'_{23}a'_{32}}_{} = a_{11}a_{22}a_{33}$	-1
$a_{12}a_{21}a_{33}$	-1	$\underbrace{a'_{12}a'_{21}a'_{33}}_{} = a_{13}a_{21}a_{32}$	-1
$a_{12}a_{23}a_{31}$	$+1$	$\underbrace{a'_{12}a'_{23}a'_{31}}_{} = a_{13}a_{22}a_{31}$	$+1$
$a_{13}a_{21}a_{32}$	$+1$	$\underbrace{a'_{13}a'_{21}a'_{32}}_{} = a_{12}a_{21}a_{33}$	$+1$
$a_{13}a_{22}a_{31}$	-1	$\underbrace{a'_{13}a'_{22}a'_{31}}_{} = a_{12}a_{23}a_{31}$	-1

$$\therefore \ \underline{\det(A') = -\det(A)}.$$

Ex) Beräkna $\begin{vmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \\ 6 & -3 & 4 \end{vmatrix}$.

$$\begin{vmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \\ 6 & -3 & 4 \end{vmatrix} \overset{\substack{\text{Bor}^* \\ (6.)}}{=} \begin{vmatrix} 2 & 1 & 3 \\ 0 & 0 & -5 \\ 0 & -6 & -5 \end{vmatrix} \overset{2.}{=} (-1) \cdot \begin{vmatrix} 2 & 1 & 3 \\ 0 & -6 & -5 \\ 0 & 0 & -5 \end{vmatrix}$$

$$= (-1) \cdot (2) \cdot (-6) \cdot (-5)$$

$$= \underline{-60}.$$

Ex) (pr 8.)

$$\overbrace{\begin{vmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \\ 5 & 7 & 3 \end{vmatrix}}^{A} \cdot \overbrace{\begin{vmatrix} 2 & -3 & 1 \\ 0 & 2 & 5 \\ 6 & 1 & 8 \end{vmatrix}}^{B} = \begin{vmatrix} 2 & 1 & 11 \\ 18 & -9 & 14 \\ 28 & 2 & 64 \end{vmatrix} = 1200$$

Sarrus regel

Sarrus regel: $(-15) \cdot (-80)$

Ex] $i \neq k$,

$$\det(a_1 \dots a_i - \lambda a_k \dots a_n) \overset{5.}{=} \det(a_1 \dots a_i \dots a_n)$$
$$+ \det(a_1 \dots -\lambda a_k \dots a_n)$$
$$\overset{4.}{=} \det(a_1 \dots a_i \dots a_n) - \lambda \cdot \underbrace{\det(a_1 \dots a_k \dots a_n)}_{=0,\ 3.}$$
$$= \det(a_1 \dots a_i \dots a_n).$$

Ex] Beräkna $\begin{vmatrix} 1 & 3 & -2 & 1 \\ 5 & 0 & 3 & 0 \\ 1 & 2 & 1 & 3 \\ 5 & 3 & 0 & 1 \end{vmatrix}$ $\left(\begin{vmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{vmatrix} \right)$

rad 2

$$= -5 \cdot \begin{vmatrix} 3 & -2 & 1 \\ 2 & 1 & 3 \\ 3 & 0 & 1 \end{vmatrix} + 0 \cdot \begin{vmatrix} 1 & -2 & 1 \\ 1 & 1 & 3 \\ 5 & 0 & 1 \end{vmatrix} - 3 \cdot \begin{vmatrix} 1 & 3 & 1 \\ 1 & 2 & 3 \\ 5 & 3 & 1 \end{vmatrix}$$

$$+ 0 \cdot \begin{vmatrix} 1 & 3 & -2 \\ 1 & 2 & 1 \\ 5 & 3 & 0 \end{vmatrix} \qquad \left(\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix} \right)$$

$$= -5 \cdot \left(3 \cdot \begin{vmatrix} -2 & 1 \\ 1 & 3 \end{vmatrix} - 0 \cdot \left| \quad \right| + 1 \cdot \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} \right)$$
$$- 3 \cdot \left(1 \cdot \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} - 1 \cdot \underbrace{\begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix}}_{=0} + 5 \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} \right)$$

$$= -5 \cdot \left(3(-2 \cdot 3 - 1 \cdot 1) + 1 \cdot (3 \cdot 1 - (-2) \cdot 2) \right)$$
$$- 3 \left(1 \cdot (2 \cdot 1 - 3 \cdot 3) + 5 \cdot (3 \cdot 3 - 1 \cdot 2) \right)$$

$$= -14.$$

Ex $A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$, $|A| \neq 0$. $\begin{pmatrix} + & - \\ - & + \end{pmatrix}$

$$A^{-1} = \frac{1}{|A|}\tilde{A} = \frac{1}{1\cdot 5 - 2\cdot 3}\cdot\begin{pmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{pmatrix}$$

$$= -1\cdot\begin{pmatrix} 5 & -3 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix}.$$

Lös: $Ax = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

Cramers regel:

$$\begin{cases} x_1 = \dfrac{\begin{vmatrix} 1 & 3 \\ -1 & 5 \end{vmatrix}}{|A|} = -1\cdot\begin{vmatrix} 1 & 3 \\ -1 & 5 \end{vmatrix} = -(1\cdot 5 - 3\cdot(-1)) = -8 \\ x_2 = \dfrac{\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}}{|A|} = -\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -(1\cdot(-1) - 1\cdot 2) = 3 \end{cases}$$

Svar: $x = \begin{pmatrix} -8 \\ 3 \end{pmatrix}$.