Ex

$$\begin{cases} 2x_1 + 5x_2 - 2x_3 = 1 \\ x_1 - 3x_2 = 7 \end{cases}$$

$a_{11} = 2$, $a_{12} = 5$,
$a_{13} = -2$.
$a_{21} = 1$, $a_{22} = -3$,
$a_{23} = 0$.
$b_1 = 1$, $b_2 = 7$.

$\underline{m = 2}$ (ekvationer)
$\underline{n = 3}$ (obekanta, x_1, x_2, x_3)

Ex

$$\begin{cases} x_1 + 2x_2 - x_3 = 2 & (1) \\ \qquad 3x_2 + x_3 = 5 & (2) \\ \qquad\qquad 4x_3 = -4 & (3) \end{cases}$$

$\underline{m = n = 3}$, <u>kvadratiskt</u> och <u>uppåt triangulärt</u> system

1°) $(3) \iff \underline{x_3 = -1}$

2°) $(2) \iff 3x_2 - 1 = 5 \iff 3x_2 = 6$
$\iff \underline{x_2 = 2}$

3°) $(1) \iff x_1 + 2 \cdot 2 - (-1) = 2$
$\iff \underline{x_1} = 2 - 1 - 4 \underline{= -3}$.

<u>Entydig lösning</u>: $(x_1, x_2, x_3) = (-3, 2, -1)$.
Systemet <u>kvadratiskt</u> $(m = n = 3)$.
Systemet <u>uppåt triangulärt</u>, ty $\underline{a_{21} = a_{31} = a_{32} = 0}$.

Ex

$$\begin{cases} x_1 - 2x_2 + x_3 = 1 \\ \quad\;\; x_2 + 2x_3 = 2 \end{cases} \qquad (*)$$

$2 = m \neq n = 3$, ej kvadratiskt,
ej triangulärt.

Välj $x_3 = s$, $s \in \mathbb{R}$. Erhåller systemet:

$$\begin{cases} x_1 - 2x_2 = 1 - s \\ \quad\;\; x_2 = 2 - 2s \end{cases}$$

Bakåtsubstitution ger lösningsmängden:

$$\begin{cases} x_1 = 1 - s + 2(2-2s) = 5 - 5s \\ x_2 \qquad\qquad\qquad\quad\; = 2 - 2s \end{cases}, \quad s \in \mathbb{R}.$$

x_3 fri variabel, x_1 och x_2 basvariabler.

Systemet $(*)$ i echelon form,
$a_{11} = 1$ och $a_{22} = 1$ pivot element
associerade med basvariablerna x_1 och x_2.

Ex 1.3.

$$\begin{cases} x_1 + 2x_2 + 3x_3 + x_4 - x_5 = 2 \\ x_3 + x_4 + x_5 = -1 \\ x_4 - 2x_5 = 4 \end{cases}$$

Echolonform, pivot element : $a_{11} = a_{23} = a_{34} = 1$.

Basvariabler : x_1, x_3, x_4

Fria variabler : $x_2 = s$, $x_5 = t$, $s, t \in \mathbb{R}$.

$$\begin{aligned} x_1 + 3x_3 + x_4 &= 2 - 2s + t \\ x_3 + x_4 &= -1 - t \\ x_4 &= 4 + 2t \end{aligned}$$

Bakåt substitution :

$$\begin{cases} x_1 = 2 - 2s + t - 3x_3 - x_4 = 13 - 2s + 8t, \\ x_3 = -1 - t - x_4 = -5 - 3t, \\ x_4 = 4 + 2t, \end{cases}$$

$\underline{s, t \in \mathbb{R}}$.

Ex

$$x_1 - 2x_2 + 3x_3 = 4 \quad (1)$$
$$-x_1 + 3x_2 + 5x_3 = 2 \quad (2)$$
$$2x_1 - x_2 + 4x_3 = 1 \quad (3)$$

1°) Eliminerar x_1 ur (2) och (3) med hjälp av (1):

$$(2) \longrightarrow (2) - (-1)\cdot(1)$$
$$(3) \longrightarrow (3) - 2\cdot(1)$$

$$x_1 - 2x_2 + 3x_3 = 4 \quad (1')$$
$$x_2 + 8x_3 = 6 \quad (2')$$
$$3x_2 - 2x_3 = -7 \quad (3')$$

2°) Eliminerar x_2 ur (3') med hjälp av (2'):

$$(3') \longrightarrow (3') - 3\cdot(2')$$

$$x_1 - 2x_2 + 3x_3 = 4 \quad (1'')$$
$$x_2 + 8x_3 = 6 \quad (2'')$$
$$-26x_3 = -25 \quad (3'')$$

3°) Bakåtsubstitution ger lösningen

$$\begin{cases} x_1 = 4 + 2x_2 - 3x_3 = -\frac{59}{26} \\ x_2 = 6 - 8x_3 = -\frac{22}{13} \\ x_3 = \frac{25}{26} \end{cases}$$

Ex

$$\begin{cases} x_1 + 2x_2 + 3x_3 + x_4 = 2 \\ 2x_1 + 4x_2 + 5x_3 + 5x_4 = 3 \\ -x_1 - 2x_2 - x_3 - 7x_4 = 4 \end{cases}$$

Räkneschema:

$$S = \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 2 \\ 2 & 4 & 5 & 5 & 3 \\ -1 & -2 & -1 & -7 & 4 \end{array}\right)$$

$$\xrightarrow{BO1^+} \begin{array}{r} \\ (2)-2\cdot(1) \\ (3)-(-1)\cdot(1) \end{array} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 2 \\ 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 2 & -6 & 6 \end{array}\right)$$

$$\xrightarrow{BO1^+} \begin{array}{r} \\ \\ (3)-(-2)\cdot(2) \end{array} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 2 \\ 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 4 \end{array}\right)$$

Echelonformen:

$$\begin{cases} x_1 + 2x_2 + 3x_3 + x_4 = 2 \\ \qquad\qquad -x_3 + 3x_4 = -1 \\ \qquad\qquad\qquad\qquad 0 = 4 \end{cases}$$

Ekvationen $0 = 4$ inkonsistent

Systemet saknar lösning.

Ex. 1.7. Systemet

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ 2x_1 - x_2 + x_3 = 2 \\ x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 2 \end{cases}$$

i echelon form:

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ -3x_2 - x_3 = 0 \\ \frac{2}{3}x_3 = 1 \\ 0 = 0 \end{cases} \iff \underset{\displaystyle \overset{\shortparallel}{S}}{\overset{\text{Echelonform}}{\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & -1 & 0 \\ 0 & 0 & \frac{2}{3} & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)}}$$

Bakåt substitution med hjälp av (BO1⁻):

$$S \xrightarrow[(3)\to\frac{3}{2}(3)]{BO3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & -1 & 0 \\ 0 & 0 & 1 & 3/2 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow[\substack{(1)-1\cdot(3) \\ (2)-(-1)(3)}]{BO1^-} \left(\begin{array}{ccc|c} 1 & 1 & 0 & -1/2 \\ 0 & -3 & 0 & 3/2 \\ 0 & 0 & 1 & 3/2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\xrightarrow[(2)\to-\frac{1}{3}(2)]{BO3} \left(\begin{array}{ccc|c} 1 & 1 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 3/2 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow[(1)-1\cdot(2)]{BO1^-} \underset{\text{Reducerad echelonform}}{\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 3/2 \\ 0 & 0 & 0 & 0 \end{array}\right)}$$

Lösning: $x_1 = 0, \; x_2 = -1/2, \; x_3 = 3/2$.

Ex

$$\begin{cases} 2x_1 - x_2 + 2x_3 = 0 \\ 4x_1 - 2x_2 - x_3 = 0 \\ 2x_1 - x_2 + x_3 = 0 \end{cases}$$

Räkneschema:

$$\begin{pmatrix} 2 & -1 & 2 \\ 4 & -2 & -1 \\ 2 & -1 & 1 \end{pmatrix} \xrightarrow{BO1^+} \begin{pmatrix} 2 & -1 & 2 \\ 0 & 0 & -5 \\ 0 & 0 & -1 \end{pmatrix} \xrightarrow{BO1^+}$$

$$\underbrace{\begin{pmatrix} 2 & -1 & 2 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{pmatrix}}_{\text{Echelon form}} \xrightarrow{BO3} \begin{pmatrix} 2 & -1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{BO1^-}$$

$$\begin{pmatrix} 2 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{BO3} \underbrace{\begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}}_{\text{Reducerad echelon form}}$$

x_1, x_3 basvariabler, x_2 fri variabel

Sätt $x_2 = s$, $s \in \mathbb{R}$.

Lösningsmängd:

$$\begin{cases} x_1 = \frac{1}{2} x_2 = s/2, \\ x_2 = s, \\ x_3 = 0, \end{cases} \quad s \in \mathbb{R}.$$