57

$A_1 = \{1,3,7\}$
$A_2 = \{2,5,6,7\}$
$A_3 = \{4\}$
$A_4 = \{3,4,7\}$
$A_5 = \{4,5\}$
$A_6 = \{3,4\}$
$A_7 = \{1,7\}$

$$A = \begin{matrix} 0&1&1&1&1&1&0 \\ 1&0&1&1&1&1&1 \\ 0&1&1&0&1&0&1 \\ 1&1&0&0&0&0&1 \\ 1&0&1&1&0&1&1 \\ 1&0&1&1&1&1&1 \\ 0&0&1&0&1&1&0 \end{matrix}$$

1° Assume $A_1, \dots, A_n$ possess a SDR.

I.e., we can choose exactly one 0 from each row, this 0 being unique. In other words, we have a set of n 0s, one in each row and one in each column $\Rightarrow$ a set of n indep. 0s.

Th 3.7: min. no. of lines covering all 0s is n.

Min. no. of lines covering $\underbrace{\text{any}}_{\text{the 0s of}}$ k col's of A is k. Because if not the total would be reduced to $k-1 + n-k = n-1$.

The no. of horiz. lines covering those 0s is $=$ no. of elements in $A_{i_1} \cup A_{i_2} \cup \dots \cup A_{i_k}$.

58 2° Assume no SDR exists.

Max no of indep. 0s is $t \leq n-1$.

t lines suffice to cover all the 0s in A.

Note. If all t vertical then some A_i's are empty!

Let $t = a + b$ a horiz., b vert.

Then the a horizontal lines cover all the zeros of $n-b$ (sets) columns, i.e. the union of $n-b$ sets contain a elements. But $a = t - b < n - b$.

$\therefore$ (*) is not satisfied.

of Hall's theorem