Exercises 6.1., Pb 4

If A is the incidence matrix of a (b, v, r, k, λ) - config. then the **complement**, meaning 0's and 1's trade places, is a
(b, v, r', k', λ') - config.

where
$$k' = v - k$$
$$r' = b - r$$
$$\lambda' = b - 2r + \lambda$$

Ex. $(7,7,3,3,1)$ - config. has a complement which is $(7,7,4,4,2)$ - config.

Let K be a $b \times v$ matrix of <u>ones</u>.
Then $K - A$ has a 0 where A has a 1

and a 1 where A has a 0.
Thus $K - A$, of dim. $b \times v$, is the incidence matrix we seek.
The row sum k' is $v - k$, the column sum r' is $b - r$. What about the

pair condition. We saw that C_{ij} $(i \neq j)$ is exactly the no. of rows where A has a 1 is site i and j simultaneously. Thus the corresponding D =

$$(K-A)'(K-A)$$

is calculated. Check that all off-diag. elements are the same. That no. is λ'.

$$(K-A)'(K-A)_{ij} = (\underset{v \times b}{K'}\,\underset{b \times v}{K} - K'A - A'K + A'A)_{ij} = b - r - r + \lambda = b - 2r + \lambda .$$

$\uparrow$ easy calc. Recall def. of K

$i \neq j$

(Incidentally, the diag. el. are $b - 2r + r = b - r$ as they should.)

$\therefore$ K − A is the incidence matrix of a $(b, v, b-r, v-k, b-2r+\lambda)$ - configuration.

6.2. Square block designs

$b = v$, $k = r$

We speak about a (v, k, λ) - design.

(1) Any row of A, the incidence matrix, contains k ones.

(2) Any column contains k ones.

(3) Any pair of columns both have ones in exactly λ rows.

(4) Any pair of rows have ones in exactly λ columns. ← Can be proved.

It is shown is 6.3. that there is an abundance of square block designs, so-called Hadamard designs.