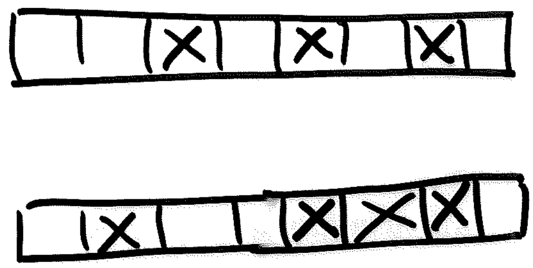
$$N(1,2) + \dots + N(n-1,n) = (n-2)!\, r_2$$

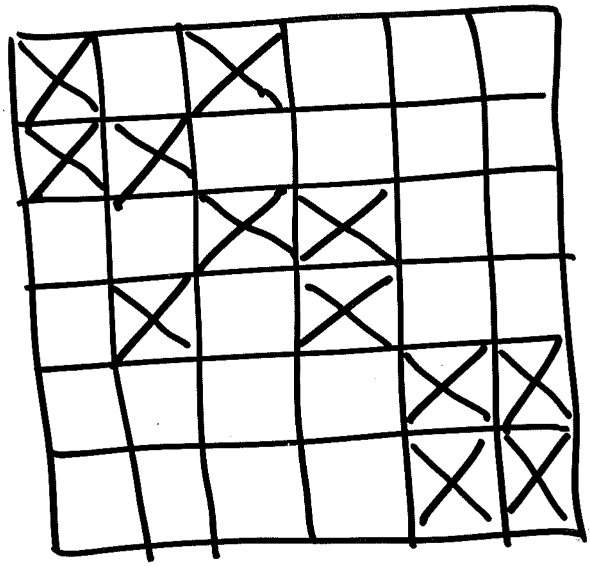


r_2 = total no. of ways to put 2 rooks in non-taking pos. on chess-board of forbidden X - positions

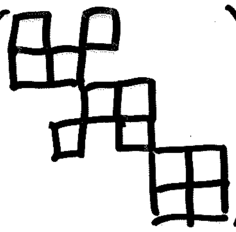
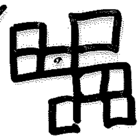
Ex. 5.8.
Constructing a Latin square.

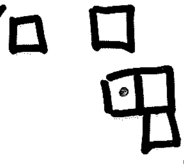
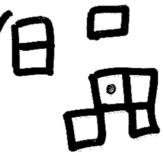
1	2	3	4	5	6
2	4	1	3	6	5

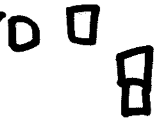
How many ways to construct line 3.



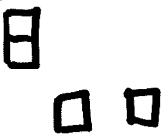
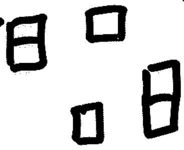
119

R() = R() $\cdot (1+4x+2x^2)$

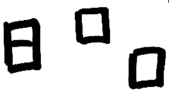
$\times$ R() + R() *

x^2 R() + x R()

$(1+x)^2$ $\quad$ $(1+2x)^2$

* x R() + R()

$(1+2x)^2$

x R() + R()

$(1+3x+x^2)(1+x)$ $\quad$ $(1+3x+x^2)(1+2x)$

$$(1+4x+2x^2)\cdot\big(x^2+2x^3+x^4+x+4x^2+4x^3$$
$$+x+4x^2+4x^3+x+3x^2+x^3$$
$$+x^2+3x^3+x^4+1+3x+x^2$$
$$+2x+6x^2+2x^3\big)$$

$$=(1+4x+2x^2)(2x^4+16x^3+20x^2+8x+1)$$
$$=4x^6+40x^5+106x^4+112x^3+54x^2+12x+1$$

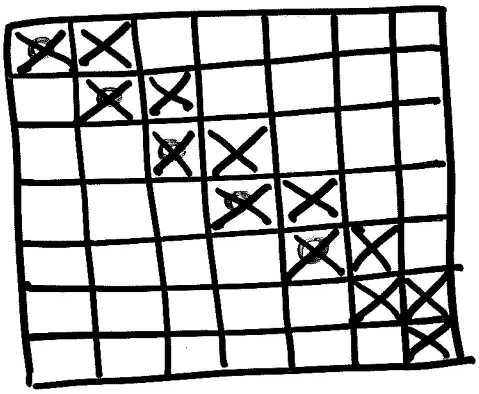
Thus

$r_6 = 4$, $r_5 = 40$, $r_4 = 106$, $r_3 = 112$,
$r_2 = 54$, $r_1 = 12$, $r_0 = 1$

Th. 5.1. gives the no. of ways to complete the Latin sq. by a third row

$$6! \cdot 1 - 5! \cdot 12 + 4!\,54 - 3!\,112$$
$$+ 2!\,106 - 1!\,40 + 4$$
$$= 720 - 1440 + 1296 - 672 + 212 - 40$$
$$+ 4 = 80.$$

Exercises 5.2, Pb 10



$n \times n$

(Here: $n = 7$)

$L_k(x)$ rook poly. of board consisting of k first crosses, reading down and to the right

$L_0(x) = 1$
$L_1(x) = 1 + x$
$L_2(x) = 1 + 2x$
$L_3(x) = 1 + 3x + x^2$
$\vdots$

$L_k(x)$ k odd

$$L_k(x) = x\,L_{k-2}(x) + L_{k-1}(x)$$

k even same recurrence rel.!

Thus

$$L_k(x) = x\,L_{k-2}(x) + L_{k-1}(x)$$

$$k \geq 2$$

Call the coeff. of $L_k(x)$ $\ell(k,r)$, $0 \leq r \leq k$.

$$L_k(x) = \ell(k,0) + \ell(k,1)x + \cdots + \ell(k,r)x^r + \cdots + \ell(k,k)x^k$$

$$\ell(k,r) = \ell(k-2,r-1) + \ell(k-1,r)$$

$$\ell(k,0) = 1, \quad \underline{\ell(k,1) = k}$$

"Lions in cages"-problem

$$g(n,k) = g(n-2,k-1) + g(n-1,k)$$

$$\underline{g(n,1) = n}, \quad g(2k-1,k) = 1$$

$\ell(2k-1,k)$? Turns out to be 1 !

Consider the chess-board with odd no. of crosses.

We know g and hence l:

$$g(n,k) = \binom{n-k+1}{k}$$

and so

$$l(k,r) = \binom{k-r+1}{r}$$

and

$$L_{2k-1}(x) = \sum_{r=0}^{2k-1} \binom{2k-r}{r} x^r .$$

The whole $n \times n$ grid has $2n-1$ crosses and so the corresp. chessboard has rook polynomial

$$L_{2n-1}(x) = \sum_{r=0}^{2n-1} \binom{2n-r}{r} x^r$$