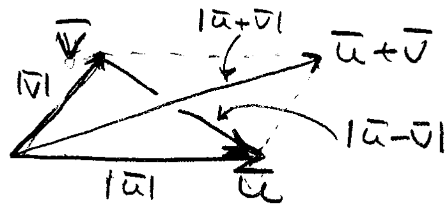
Demonstrationer i FDA, vecka 3

1. a) Visa att om $\bar{u}$ och $\bar{v}$ är vektorer i $\mathbb{R}^n$ så gäller: $|\bar{u}+\bar{v}|^2+|\bar{u}-\bar{v}|^2=2(|\bar{u}|^2+|\bar{v}|^2)$.

$$\begin{cases} |\bar{u}+\bar{v}|^2=(\bar{u}+\bar{v})\cdot(\bar{u}+\bar{v}) = \bar{u}\cdot\bar{u}+\bar{u}\cdot\bar{v}+\bar{v}\cdot\bar{u}+\bar{v}\cdot\bar{v} \\ \qquad = |\bar{u}|^2+2\bar{u}\cdot v+|\bar{v}|^2 \\ |\bar{u}-\bar{v}|^2=(\bar{u}-\bar{v})\cdot(\bar{u}-\bar{v}) = \bar{u}\cdot\bar{u}-\bar{u}\cdot\bar{v}-\bar{v}\cdot\bar{u}+\bar{v}\cdot\bar{v} \\ \qquad = |\bar{u}|^2-2\bar{u}\cdot\bar{v}+|\bar{v}|^2 \end{cases}$$

Addition ger: $|\bar{u}+\bar{v}|^2+|\bar{u}-\bar{v}|^2=2(|\bar{u}|^2+|\bar{v}|^2)$.

Geometrisk tolkning i $\mathbb{R}^2$: "Summan av kvadraterna av diagonalerna = summan av kvadraterna av sidorna"
Parallellogramidentiteten



b) Om $\bar{u}$ och $\bar{v}$ är vektorer i $\mathbb{R}^3$ så gäller: $(\bar{u}+\bar{v})\times(\bar{u}-\bar{v}) = 2\bar{v}\times\bar{u}$.

$$\begin{aligned} \underline{(\bar{u}+\bar{v})\times(\bar{u}-\bar{v})} &= (\bar{u}+\bar{v})\times\bar{u}-(\bar{u}+\bar{v})\times\bar{v} && \text{(distr. lagen)} \\ &= \bar{v}\times(\bar{u}+\bar{v})-\bar{u}\times(\bar{u}+\bar{v}) && \text{(antikommutativ)} \\ &= \bar{v}\times\bar{u}+\underbrace{\bar{v}\times\bar{v}}_{\bar{0}}-\underbrace{\bar{u}\times\bar{u}}_{\bar{0}}-\bar{u}\times\bar{v} && \text{(distr. lagen)} \\ &= \bar{v}\times\bar{u}+\bar{v}\times\bar{u} && \text{(antikommutativ)} \\ &= \underline{2(\bar{v}\times\bar{u})}. \; \square \end{aligned}$$

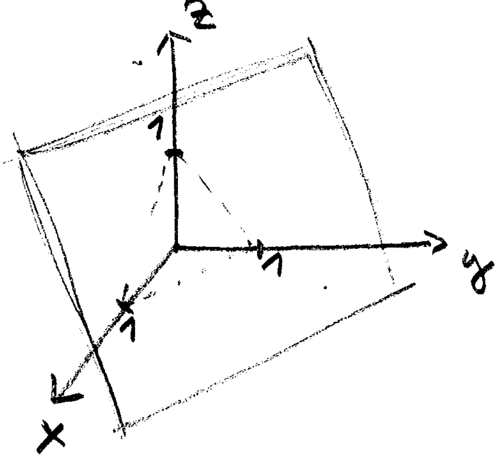
2. Åskådliggör funktionerna: a) $z = 1-x-y$,

b) $z = \sqrt{1-x^2-y^2}$,

som funktionsgrafer och nivåkurvor.

a) $z = 1-x-y \iff x+y+z-1=0$, ett plan i $\mathbb{R}^3$.

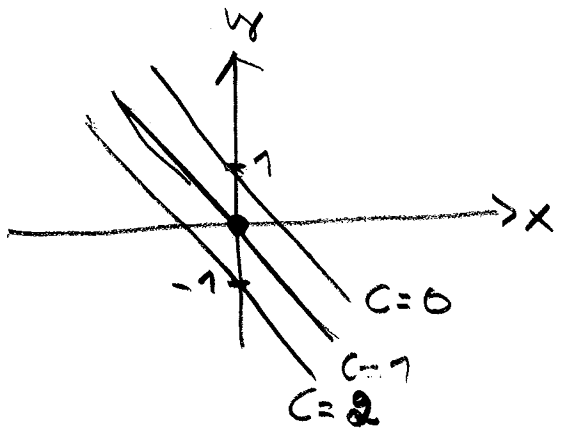
$$\begin{cases} \text{Planet skär } xy\text{-planet i linjen } x+y=1 \\ \text{— " — } yz\text{-planet i linjen } y+z=1 \\ \text{— " — } xz\text{-planet i linjen } x+z=1 \end{cases}$$



Punkterna: $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ ligger i planet

Nivåkurvorna i xy-planet lösningar till ekvationen

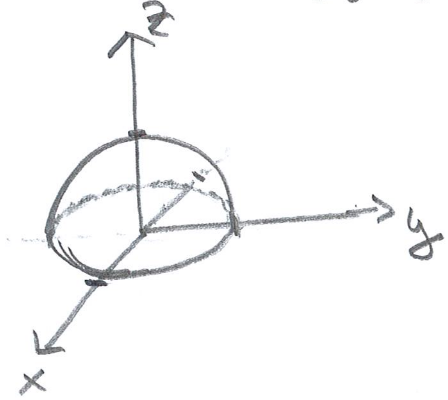
$1-x-y = c$, dvs. linjerna $y = -x+(1-c)$, $c \in \mathbb{R}$



2. b) $z = \sqrt{1-x^2-y^2} \iff z^2 = 1 - x^2 - y^2,\ z \geq 0$

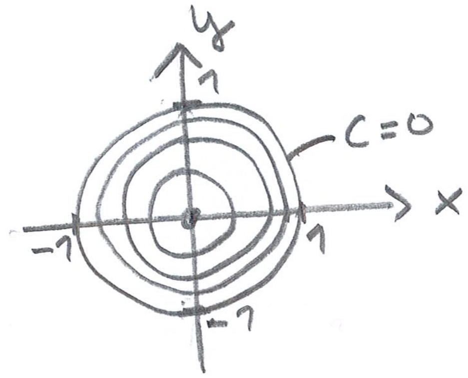
$\iff x^2 + y^2 + z^2 = 1,\ z \geq 0.$

Vilket är övre halvan av enhetssfären i $\mathbb{R}^3$:



Nivåkurvorna i xy-planet lösningar till ekvation $\sqrt{1-x^2-y^2} = C \iff x^2 + y^2 = 1 - C^2$,

$0 \leq C \leq 1$.

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In[1]:= f[x_, y_] := Sqrt[1 - x^2 - y^2]

In[2]:= ContourPlot[f[x, y], {x, -1, 1}, {y, -1, 1}]
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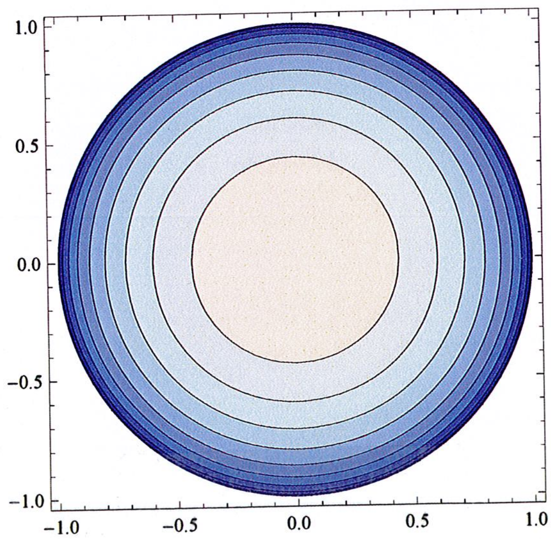
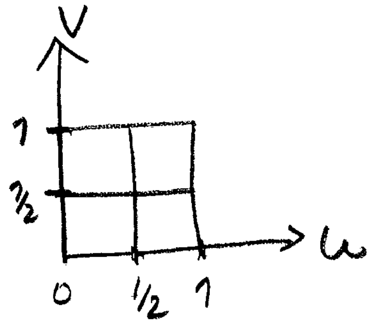
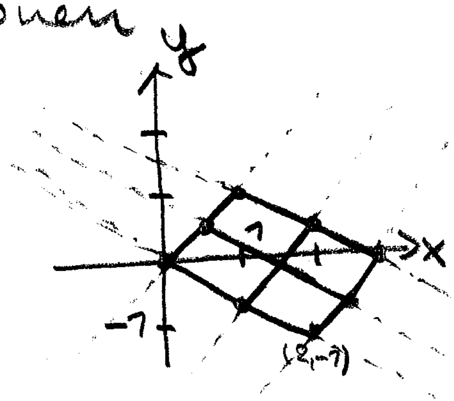
Out[2]=



3. Vilken form får kvadraten $0 \le u \le 1$, $0 \le v \le 1$, i uv-planet efter deformationen

a) $\begin{cases} x = u + 2v, \\ y = u - v. \end{cases}$





linjen $u=0$ avbildas på linjen $y = -\frac{x}{2}$, $\begin{cases}(0,0) \to (0,0) \\ (0,1) \to (2,-1)\end{cases}$

— " — $u=1$ —— " —— $y = -\frac{x}{2} + \frac{1}{2}$, $\begin{cases}(1,0) \to (1,1) \\ (1,1) \to (3,0)\end{cases}$

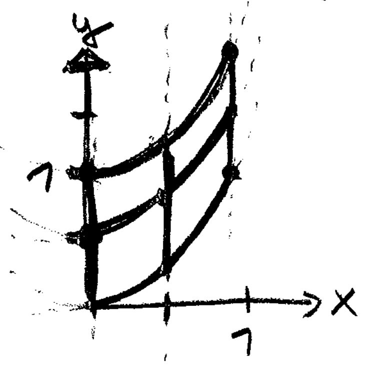
— " — $u = \frac{1}{2}$ —— " —— $y = y = -\frac{x}{2} + \frac{3}{4}$, $\begin{cases}(1/2,0) \to (\frac{1}{2},\frac{1}{2}) \\ (1/2,1) \to (\frac{5}{2}, -\frac{1}{2})\end{cases}$

linjen $v=0$ avbildas på linjen $y = x$, $\begin{cases}(0,0) \to (0,0) \\ (1,0) \to (1,1)\end{cases}$

— " — $v=1$ —— " —— $y = x - 3$, $\begin{cases}(0,1) \to (2,-1) \\ (1,1) \to (3,0)\end{cases}$

linjen $v = \frac{1}{2}$ avbildas på linjen $y = x - \frac{3}{2}$, $\begin{cases}(0,1/2) \to (1,-\frac{1}{2}) \\ (1,1/2) \to (2,\frac{1}{2})\end{cases}$

b) $\begin{cases} x = u \\ y = u^2 + v \end{cases}$



linjen $v=0$ avbildas på $y = x^2$, $\begin{cases}(0,0) \to (0,0) \\ (1,0) \to (1,1)\end{cases}$

linjen $v = 1$ —— " —— $y = x^2 + 1$, $\begin{cases}(0,1) \to (0,1) \\ (1,1) \to (1,2)\end{cases}$

linjen $v = \frac{1}{2}$ —— " —— $y = x^2 + \frac{1}{2}$ $\begin{cases}(0,1/2) \to (0,1/2) \\ (1,1/2) \to \end{cases}$

$u=0$ avbildas på $x=0$

$u = \frac{1}{2}$ —— " —— $x = \frac{1}{2}$

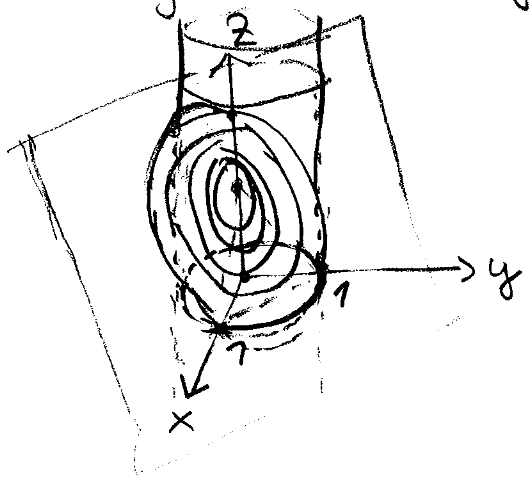
$u = 1$ —— " —— $x = 1$

4. Vilken form får cirkeln $u^2+v^2 \leq 1$ i uv-planet efter deformationen

a) $\begin{cases} x = u \\ y = v \\ z = 1-u-v \end{cases}$ $\therefore$ $x+y+z = u+v+(1-u-v) = 1$

$\therefore$ Cirkelområdet $u^2+v^2 \leq 1$ avbildas på ett område som ligger i planet $x+y+z=1$.

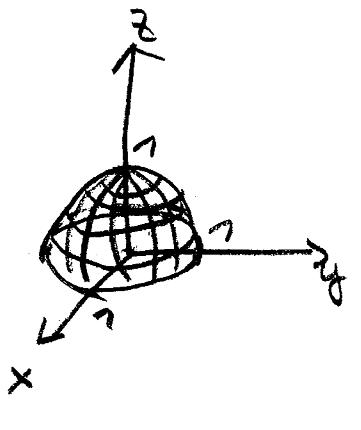
Området i planet blir ett elliptiskt område som cylindern $x^2+y^2=1$ skär ut ur planet.



b) $\begin{cases} x=u \\ y=v \\ z=\sqrt{1-u^2-v^2} \end{cases}$ $\therefore$ $z^2+x^2+y^2 = 1-u^2-v^2+u^2+v^2 = 1, \ z \geq 0$

$\therefore$ $x^2+y^2+z^2=1, \quad z \geq 0$

Cirkelskivan $u^2+v^2 \leq 1$ avbildas på övre halvan av enhetssfären i xyz-rummet:



cirkeln $u^2+v^2=r^2$, $r \leq 1$, avbildas på kurvan $(r\cos\theta, r\sin\theta, \sqrt{1-r^2})$, $0 \leq \theta < 2\pi$, som är en cirkel på sfärens yta

4.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In[2]:= x[r_, t_] := r Cos[t]; y[r_, t_] := r Sin[t]; z[r_, t_] := 1 - x[r, t] - y[r, t]

In[3]:= ParametricPlot3D[{x[r, t], y[r, t], z[r, t]}, {r, 0, 1}, {t, 0, 2 Pi}]
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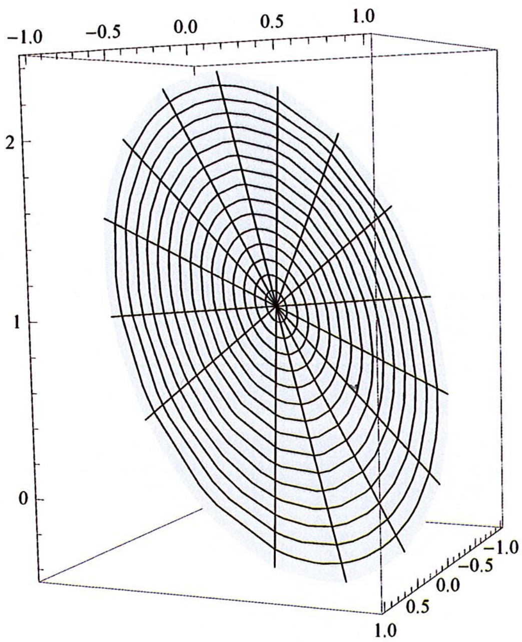
Out[3]=

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In[4]:= x[r_, t_] := r Cos[t]; y[r_, t_] := r Sin[t];
        z[r_, t_] := Sqrt[1 - x[r, t]^2 - y[r, t]^2]

In[5]:= ParametricPlot3D[{x[r, t], y[r, t], z[r, t]}, {r, 0, 1}, {t, 0, 2 Pi}]
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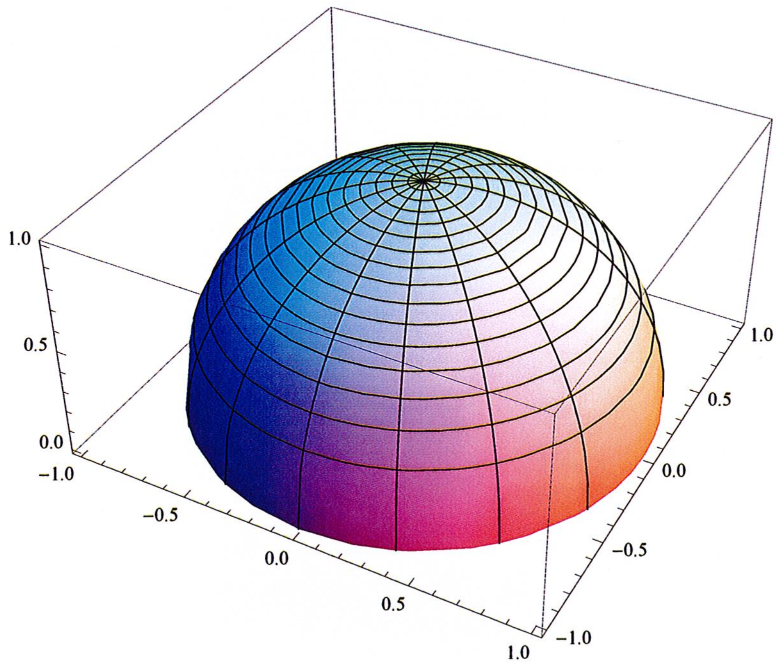
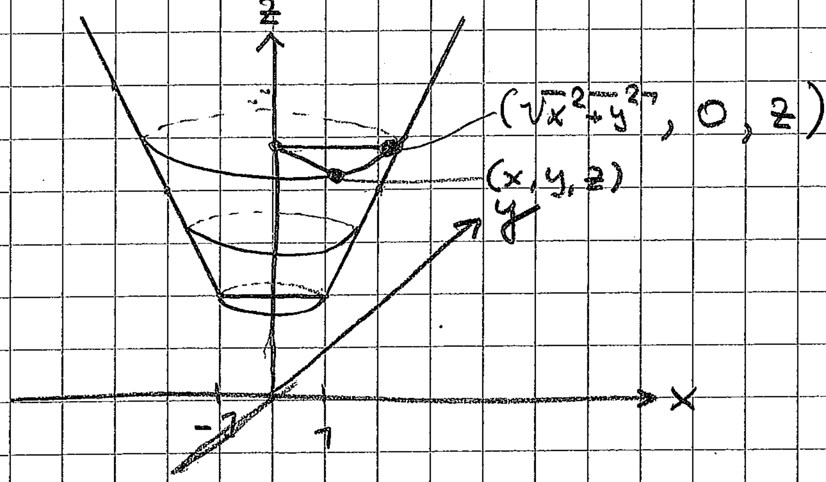
Out[5]=



5 Funktionskurvan $z = f(x) = |x+1| + |x-1|$ roterar kring z-axeln. Bestäm ekvationen för den därvid alstrade funktionsytan $z = g(x,y)$. Skissera ytan.

$$z = f(x) = \begin{cases} -2x, & x \le -1 \\ 2, & -1 < x \le 1 \\ 2x, & 1 < x \end{cases}$$



$\therefore$ (x, y, z) punkt på ytan $\Rightarrow$ $(\sqrt{x^2+y^2}, 0, z)$ punkt på ytan

$$\Rightarrow z = f(\sqrt{x^2+y^2}) = g(x,y)$$

$\therefore$ $\underline{z = g(x,y)} = |\sqrt{x^2+y^2}+1| + |\sqrt{x^2+y^2}-1|$

$$= \underline{1 + \sqrt{x^2+y^2} + |\sqrt{x^2+y^2} - 1|}$$