

The Rozov exercises

M.2.1.

$$\begin{aligned}x' &= 1 - xy^A \\ y' &= Bxy^A - Cy\end{aligned}$$

$$x, y > 0, A, B, C > 0$$

M.2.2.

$$\begin{aligned}y_1' &= y_1 \left(-E_1 + V_0(C - y_1 - y_2) - \frac{y_2}{R+y_1} \right) \\ y_2' &= \left(-E_2 + \frac{y_1}{R+y_1} \right) y_2\end{aligned}$$

$$y_i \geq 0, 0 < E_i < 1, R > 0, V_0 > 0, C > 0.$$

M.2.3.

$$\begin{aligned}y_1' &= \frac{1}{E} \left(y_2 - \frac{y_1^3}{3} + y_1 \right) \\ y_2' &= -E y_1\end{aligned}$$

$$0 < E \leq 1.$$

M.2.4

$$\begin{aligned}y_1' &= A - B y_1 - y_1 + y_1^2 y_2 \\ y_2' &= B y_1 - y_1^2 y_2\end{aligned}$$

$$A, B > 0.$$

M.2.5

$$\begin{aligned}y_1' &= y_1(\nu - 1 + y_2) \\ y_2' &= \nu y_2 - (y_1^2 + y_2^2)\end{aligned}$$

$$\nu \geq 0.$$

M.2.6

$$\begin{aligned}y_1' &= A - y_1 y_2 \\ y_2' &= y_1 y_2 - \frac{y_2}{B+y_2}\end{aligned}$$

$$y_1, y_2 > 0, A, B > 0.$$

M.2.7

$$\begin{aligned}y_1' &= Ay_2 + E_1y_1 - y_1^3 - y_1y_2^2 \\y_2' &= -Ay_1 + E_2y_2 - y_2^3 - y_1^2y_2\end{aligned}$$

$$A, E_1, E_2 \geq 0.$$

M.2.8

$$\begin{aligned}y_1' &= By_2(1 - y_1) - Dy_1 \\y_2' &= By_2(1 - y_1(1 + A) + (y_2 - A)^2) + C \cdot B\end{aligned}$$

$$A, B, C, D \geq 0.$$

M.2.9

$$\begin{aligned}y_1' &= y_1(A_1y_2 - B_1) \\y_2' &= y_2(A_2y_1 - B_2)\end{aligned}$$

$$A_i, B_i \geq 0.$$

M.2.10

$$\begin{aligned}y_1' &= \mu + y_1 + y_2 - y_1^3/3 \\y_2' &= C(A - y_1 - By_2)\end{aligned}$$

$$0 < B, C < 1.$$

M.3.1.

$$\begin{aligned}y_1' &= y_1((3 - 4\mu) - y_1 - y_2 - y_3) \\y_2' &= y_2(-E_2 + y_1 + 2\mu y_2) \\y_3' &= y_3(-E_3 + y_2)\end{aligned}$$

$$E_i, \mu, y_i \geq 0.$$

M.3.2.

$$\begin{aligned}y_1' &= y_1 \left(-E_1 + \alpha(C - \sum_{i=1}^3 y_i) - \frac{y_2}{y_1 + R} \right) \\y_2' &= y_2 \left(-E_2 + \frac{y_1}{y_1 + R} - \frac{y_3}{y_2 + R} \right) \\y_3' &= y_3 \left(-E_3 + \frac{y_2}{R + y_2} \right)\end{aligned}$$

$$y_i \geq 0, \alpha, C, K > 0, 0 < E_i < 1.$$

M.3.3.

$$\begin{aligned}y_1' &= y_1(E_1 - y_2) \\y_2' &= y_2(-E_2 + y_1 - y_3) \\y_3' &= y_3(-E_3 + \mu y_2)\end{aligned}$$

$$y_i \geq 0, E_i, \mu > 0.$$

M.3.4.

$$\begin{aligned}y_1' &= E_1 - y_1y_2 \\y_2' &= y_2(-E_2 + y_1 - y_3) \\y_3' &= y_3(-E_3 + \mu y_2)\end{aligned}$$

$$y_i \geq 0, E_i, \mu > 0.$$

M.3.5.

$$\begin{aligned}y_1' &= y_1(E_1 - y_2) \\y_2' &= y_2(-E_2 + y_1 - y_3) \\y_3' &= -E_3 + \mu y_2 y_3\end{aligned}$$

$$y_i \geq 0, E_i, \mu > 0.$$

M.3.6.

$$\begin{aligned}y_1' &= -My_1 + y_2 y_3 \\y_2' &= -My_2 + y_1(y_3 - A) \\y_3' &= 1 - y_1 y_2\end{aligned}$$

$$A, M \geq 0.$$

M.3.7.

$$\begin{aligned}y_1' &= -y_2 - y_3 \\y_2' &= y_1 + Ay_2 \\y_3' &= F - My_3 + y_1 y_3\end{aligned}$$

$$A, F, M \geq 0.$$

M.3.8.

$$\begin{aligned}y_1' &= -S(y_1 - y_2) \\y_2' &= Ry_1 - y_2 - y_1 y_3 \\y_3' &= y_1 y_2 - By_3\end{aligned}$$

M.3.9.

$$\begin{aligned}y_1' &= (\phi - 1)y_1 - y_2 + y_1 y_3 \\y_2' &= y_1 + (\phi - 1)y_2 + y_2 y_3 \\y_3' &= \phi y_3 - (y_1^2 + y_2^2 + y_3^2)\end{aligned}$$

$$0 \leq \phi \leq 1.$$

M.3.10.

$$\begin{aligned}y_1' &= y_1(\alpha - \gamma y_3) \\y_2' &= \lambda y_1 - \beta y_2 \\y_3' &= \mu y_2 - \gamma y_3\end{aligned}$$

$$\alpha, \beta, \gamma, \lambda, \mu > 0.$$

M.3.11.

$$\begin{aligned}y_1' &= A - (B + 1)y_1 + y_1^2 y_2 + y_2 y_3 \\y_2' &= By_1 - y_1^2 y_2 - y_2 y_3 \\y_3' &= R(y_1 - y_2 y_3)\end{aligned}$$

$$A, B, R > 0.$$

M.3.12.

$$\begin{aligned}y_1' &= A + C \cos(W y_3) - (B + 1)y_1 + y_1^2 y_2 \\y_2' &= By_1 - y_1^2 y_2 \\y_3' &= 1\end{aligned}$$

$$A, B, C, W > 0.$$

M.3.13.

$$\begin{aligned} y_1' &= y_1 - \frac{D}{2}y_2 + y_2(y_3 + y_1^2) \\ y_2' &= \frac{D}{2}y_1 + y_2 + y_1(3y_3 - y_1^2) \\ y_3' &= -2y_3(M + y_1y_2) \end{aligned}$$

$$D, M > 0.$$

M.3.14.

$$\begin{aligned} y_1' &= y_1 - 2y_2^2 - Dy_2 + y_3 \\ y_2' &= Dy_1 + 2y_1y_2 + y_2 \\ y_3' &= -2y_3(y_1 + M) \end{aligned}$$

$$D, M \geq 0.$$

M.3.15.

$$\begin{aligned} y_1' &= 2y_1 - 2y_1(y_1 + y_2) - y_1y_2(\cos y_3 + C_2 \sin y_3) \\ y_2' &= 2y_2 - 2y_2\left(2y_1 + \frac{3}{4}y_2\right) - 2y_1y_2(\cos y_3 - C_2 \sin y_3) - 2K^2y_2 \\ y_3' &= C_2(2y_1 - (1/2)y_2) + 2C_1K^2 + \sin y_3(2y_1 + y_2) + C_2 \cos y_3(2y_1 - y_2) \end{aligned}$$

M.3.16.

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= (\sin y_1 \cos y_1)y_3^2 - \sin y_1 - Ay_2 \\ y_3' &= K(\cos y_1 - R) \end{aligned}$$

$$K > 0, 0 < R < 1.$$

M.3.17.

$$\begin{aligned} y_1' &= Hy_1 + y_2 + Ey_3 \\ y_2' &= -y_1 \\ y_3' &= \frac{1}{C} \{-A(y_1 + y_1^3 + y_3) + th(B(1 + 4y_3 - 16y_1))\} \end{aligned}$$

M.3.18.

$$\begin{aligned} y_1' &= \alpha(y - xy + x - Gx^2) \\ y_2' &= \frac{1}{\alpha}(-y - xy + Fz) \\ y_3' &= \mu(x - \delta xz - z) \end{aligned}$$

M.3.19.

$$\begin{aligned} x' &= y \\ y' &= -Dy + \frac{1}{2}x(1 - x^2) + F \cos z \\ z' &= W \end{aligned}$$

M.3.20.

$$\begin{aligned} x' &= y \\ y' &= -Ky - x^3 + B \cos z \\ z' &= 1 \end{aligned}$$

M.3.21.

$$\begin{aligned}y_1' &= -Ay_1 + y_2 + By_2y_3 \\y_2' &= -y_1 - Cy_2 + Dy_1y_3 \\y_3' &= \alpha y_3 - Ey_1y_2\end{aligned}$$

$$A, B, C, D, E > 0.$$

M.3.22.

$$\begin{aligned}y_1' &= 1 - by_1 - y_1y_2^2 - qy_1y_2 + y_3 \\y_2' &= a(y_1y_2^2 - y_2 + d) \\y_3' &= c(qy_1y_2 - y_3)\end{aligned}$$

$$a, b, c, d, q > 0.$$

M.3.23.

$$\begin{aligned}y_1' &= y_1(y_1 - B)(1 - y_1) - y_1y_3 - A(y_1 - y_2) \\y_2' &= y_2(y_2 - B)(1 - y_2) - y_2y_3 - A(y_2 - y_1) \\y_3' &= y_3(y_1 + y_2 - C)\end{aligned}$$

M.4.1.

$$\begin{aligned}y_1' &= -y_2 - y_3 \\y_2' &= y_1 + Ay_2 + y_4 \\y_3' &= B + y_1y_3 \\y_4' &= -Cy_3 + Dy_4\end{aligned}$$

$$A, B, C, D > 0.$$

M.4.2.

$$\begin{aligned}y_1' &= y_1 \left(-m_1 + a_0(C - \sum_{i=1}^4 y_i) - a_1y_2 \right) \\y_2' &= y_2(-m_2 + a_1y_1 - a_2y_3) \\y_3' &= y_3(-m_3 + a_2y_2 - a_3y_4) \\y_4' &= y_4(-m_4 + a_3y_3)\end{aligned}$$

$$C, m_i, a_i > 0.$$

M.4.3.

$$\begin{aligned}y_1' &= y_1 (-E_1 - A_1y_2/(R + ay_1) + B_1y_0/(R + by_0)) \\y_2' &= y_2 (-E_2 + A_2y_1/(R + ay_1)) \\y_3' &= y_3 (-E_3 - A_3y_4/(R + ay_3) + B_3y_0/(R + by_0)) \\y_4' &= y_4 (-E_4 + A_4y_3/(R + ay_3))\end{aligned}$$

$$E_i, A_i, B_i, a, b, M, R \geq 0.$$

M.4.4.

$$\begin{aligned}X_{1,2}' &= A_{1,2} - (B_{1,2} + 1)X_{1,2} + X_{1,2}^2Y_{1,2} + C_x(X_{2,1} - X_{1,2}) \\Y_{1,2}' &= B_{1,2}X_{1,2} - X_{1,2}^2Y_{1,2} + C_y(Y_{2,1} - Y_{1,2})\end{aligned}$$

$$A_1, A_2, B_1, B_2, C_x, C_y \geq 0.$$