Cross hedging and backward stochastic differential equations

P. Imkeller
Humboldt-Universität zu Berlin

Abstract

A financial market model is considered on which agents (e.g.insurers) are subject to an exogenous, for instance climate-based, financial risk, which they trade by issuing a risk bond. Buyers of the bond are able to invest in a market asset correlated with the exogenous risk. We investigate their optimal investment problem, and calculate bond prices using utility indifference. This hedging concept is interpreted by means of martingale optimality, and solved with backward stochastic differential equations (BSDE) and Malliavin's calculus tools.

In this short course we will develop BSDE with drivers of quadratic growth from the context of this utility maximization problem, starting with martingale optimality. The basic theory of existence and uniqueness of solutions of BSDE with Lipschitz continuous drivers will be presented. A more general (weaker) notion of solutions of BSDE, so-called *measure solutions*, will be considered, investigated by tools of weak convergence of probability measures, and used to extend the realm of solutions to drivers of quadratic growth. Finally BSDE theory and Malliavin's calculus will be seen to allow a purely stochastic approach of the concept of cross hedging, containing an explicit extension of the Black-Scholes delta hedge to incomplete markets.