Riemann - Stieltjes integrals and applications in mathematical finance

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Abstract

We study the class $\mathcal{A}_{RS}(B^H)$ of all càdlàg and adapted $\big[$with respect to filtration of fractional Brownian motion $B^H, H \in (\frac{1}{2}, 1)\big]$ stochastic process $u = \{u_t\}_{t\in[0,T]}$ such that Riemann - Stieltjes integral

$$(\mathrm{RS}) - \int_0^T u_t dB_t^H$$

exists almost surely. According to Young integration theory, class $\mathcal{A}_{RS}(B^H)$ contains all stochastic processes with trajectories of α - Hölder continuous for some $\alpha > 1 - H$. It turns out that all processes of the form $u_t = f_-^{'}(B_t^H)$ belong to class $\mathcal{A}_{RS}(B^H)$ for a convex function f. Moreover the classical change of variable formula holds. This suggests that fractional Brownian motion shares some common properties with continuous functions of bounded variation. On the other hand, we show a property on running maximum of continuous functions of bounded variation that cannot apply to fractional Brownian motion.

As financial application, we study an asymptotic hedging problem for European options with convex payoff in fractional Black - Scholes market with proportional transaction costs. The talk is based on articles [1], [2] and [3].

References

[1] Azmoodeh, E., Mishura, Y., and Valkeila, E. (2010). *On hedging European options in geometric fractional Brownian motion market model*. Statistics & Decisions, forthcoming.

[2] Azmoodeh, E., (2010). *On the fractional Black-Scholes market with transaction costs*. Submitted.

[3] Azmoodeh, E., Tikanmäki, H., Valkeila, E. (2010). *When fractional Brownian motion does not behave as a continuous function with bounded variation?* Submitted.