Päivölä, June 1-5

Topics in Random Graphs

by M. Bloznelis
http://www.mif.vu.lt/∼bloznelis

Random graphs and large networks is an area of active research. It lies on the borderline of combinatorics and probability theory. Statistical physics, social statistics, computer science,... are among many areas of applications here and a rich source of problems.

In this very short course I shall address several questions, which illustrate the methods/approaches used in the area as well as phenomena studied.

In the first part I introduce binomial random graph and discuss: subgraph containment problem - how many edges you need to throw chaotically in order to force the appearance with a high probability (whp) of your favorite pattern; connectivity threshold - how many edges ensure the connectivity property whp; birth of the giant component - how many edges ensure the existence of a large connected cluster whp [1]. On the way I will explain the methods used: counting via expectations (the first and second moment method), counting of trees directly and by using branching process approximation of a local search.

The second part of my lectures will focus on random graphs that model real networks. Firstly, I shall explain two nice properties of power law graphs: ultra short typical distances between nodes (small world) [3] and polynomial clique number (the number of vertices of the largest complete subgraph) [2]. Secondly I will introduce edge dependence characteristics (clustering, assortativity coeff.) and briefly discuss clustering properties of random intersection graphs [5]. Finally, if time permits, I shall explain how preferred attachment principle leads to a power law degree distribution [4].

During my lectures I shall explain concepts used. For example, you are not required to know what is a connected graph - this will be explained as well as other very few notions of graph theory that will be needed.

References

[1] S. Janson, T. Łuczak, and A. Ruciński, *Random Graphs*, Wiley, New York, 2001.

[2] S. Janson, T. Łuczak and I. Norros, Large cliques in a power-law random graph, *J. Appl. Probab.* **47** (2010), 1124–1135.

[3] I. Norros and H. Reittu, On the power - law random graph model of massive data networks, *Performance Evaluation* **55** (2004), 3–23.

[4] Móri, T. F. (2002). On random trees. Studia Sci. Math. Hungar. 39, 143–155.

[5] M. Bloznelis, Degree and clustering coefficient in sparse random intersection graphs, The Annals of Applied Probability 23 (2013), 1254–1289.