Riemann-Stieltjes integrals with respect to fractional Brownian motion and applications

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Abstract

In this dissertation we study Riemann-Stieltjes integrals with respect to (geometric) fractional Brownian motion, its financial counterpart and its application in estimation of quadratic variation process. From the point of view of financial mathematics, we study the fractional Black-Scholes model in continuous time.

We show that the classical change of variable formula with convex functions holds for the trajectories of fractional Brownian motion. Putting it simply, all European options with convex payoff can be hedged perfectly in such pricing model. This allows us to give new arbitrage examples in the geometric fractional Brownian motion case. Adding proportional transaction costs to the discretized version of the hedging strategy, we study an approximate hedging problem analogous to the corresponding discrete hedging problem in the classical Black-Scholes model. Using the change of variables formula result, one can see that fractional Brownian motion model shares some common properties with continuous functions of bounded variation. we also show a representation for running maximum of continuous functions of bounded variations such that fractional Brownian motion does not enjoy this property.