

Independence properties of the Matsumoto-Yor type and characterization of Kummer, gamma and beta distributions

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We define Letac-Wesolowski-Matsumoto-Yor functions as decreasing functions from $(0, \infty)$ onto $(0, \infty)$ with the following property: there exist independent, positive random variables X and Y such that the variables $f(X+Y)$ and $f(X) - f(X+Y)$ are independent. We prove that, under additional assumptions, there are essentially four such functions. The first one $f(x) = 1/x$ has been treated by Matsumoto-Yor (2001) and Letac and Wesolowski (2000) and is referred in the literature as *the Matsumoto-Yor property*. In that case, the law of X is generalized inverse Gaussian while Y is gamma-distributed. As for the three other cases, under the weak assumption that X and Y have densities functions whose logarithm is locally integrable, we prove that the distribution of (X, Y) is unique. This leads to Kummer, gamma and beta distributions and as a byproduct we obtain a new relation of convolution involving gamma distributions and Kummer distributions of type 2.