This is a list of more relevant changes/corrections made in the second edition of

**HANDBOOK OF BROWNIAN MOTION - FACTS AND FORMULAE.**

published year 2002 by Birkhäuser Verlag. These are incorporated into the corrected reprint of the second edition published year 2015.

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1. Chapter II item 10 p. 18 l. -1: “solutions” should be “positive solutions”.
2. Chapter II item 27 p. 32 l. -1: \( E_x(A_t; t < \zeta) \) should be \( E_x(A_t^n; t < \zeta) \).

3. Chapter III: A new item discussing Zvonkin’s condition added after item 22 p. 47.
4. Chapter IV: Two new items discussing perpetual integral functionals added after item 33 p. 70.
5. Chapter IV item 15 p. 59 l. -6: Sentence “The limit is in the sense of weak convergence of measures on the function space \( E^+ \cap \{ \zeta = l \} \)” changed to “The limit is in the sense of the convergence of finite dimensional distributions.”
6. Chapter VI item 1 p. 103 l. 6: Sentence “Assume also that \( F \) and \( g \) are bounded.” changed to “Assume that \( g \) is bounded and that \( F \) is bounded and Hölder continuous.”
7. Appendix 1 item 9 p. 124: Added text at the end: “The expression for the density is valid for all nonnegative values on \( c \) and \( \gamma \); also when \( \gamma c > 1 \). We remark that the densities in No. 7 and No. 8 can be obtained from the density above by letting \( c \to 0 \) and \( \gamma \to 0 \), respectively.”
8. Appendix 1 item 11 p. 125: Added text at the end: “The expression for the density is valid for all nonnegative values on \( c \) and \( \gamma \); also when \( 4 \gamma c > 1 \).”
9. Appendix 1 item 12 p. 126: Following corrections:

   **Speed measure:**
   \[
   m(dx) = \begin{cases} 
   4\beta \, dx, & x > 0, \\
   4(1 - \beta) \, dx, & x < 0. 
   \end{cases}
   \]

   **Scale function:**
   \[
   s(x) = \begin{cases} 
   \frac{x}{2\beta}, & x \geq 0, \\
   \frac{x}{2(1 - \beta)}, & x \leq 0. 
   \end{cases}
   \]

   **Green function:**
   \[
   G_\alpha(x, y) = \frac{e^{-|x-y|\sqrt{2\alpha}} - e^{-(|x|+|y|)\sqrt{2\alpha}}}{2\sqrt{2\alpha}(1 + (2\beta - 1) \text{sgn}(x \wedge y))} + \frac{e^{-(|x|+|y|)\sqrt{2\alpha}}}{2\sqrt{2\alpha}}.
   \]
Wronskian: $w_\alpha = 2\sqrt{2}\alpha$.

Transition density w.r.t. $m$:

$$p(t; x, y) = \frac{e^{-\frac{(x-y)^2}{2t}} - e^{-\frac{|x+y|^2}{2t}}}{2\sqrt{2}\pi t (1 + (2\beta - 1) \text{sgn}(x \cdot y))} + \frac{e^{-\frac{|x+y|^2}{2t}}}{2\sqrt{2}\pi t}. $$

10. Appendix 1 item 13 p. 127: “$\Upsilon = \sqrt{1 - 4\gamma c}$” should be “$\Upsilon = \sqrt{1 - \gamma c}$”. Added text at the end: “The expression for the density is valid for all nonnegative values of $c$ and $\gamma$; also when $\gamma c > 1$.”

11. Appendix 1 item 21 p. 133: Added: “Wronskian: $w_\alpha = 1$.”

12. Appendix 1 item 24 p. 137: the transition density in case $\gamma > 0$ is as follows:

$$p(t; x, y) = \frac{1}{2\sqrt{2\pi}} \frac{\sqrt{|\gamma|}}{\text{sh}(\sqrt{|\gamma|}t)} \exp\left(-\frac{|\gamma| (x^2 + y^2)}{2} - \frac{(x^2 + y^2)|\gamma| \text{ch}(|\gamma|t) - 2xy|\gamma|}{2\text{sh}(\sqrt{|\gamma|}t)}\right).$$

13. Appendix 1 item 25 p. 139 l.12: the sentence “For $\nu = 0$ this is ... of freedom.” should be “For $\nu = 0$ and $\gamma = 1/2$ this is the so-called Rayleigh distribution, for $\nu = 1/2$ and $\gamma = 1/2$ we have the Maxwell distribution and, in general, for $\nu = n/2 - 1$, $n = 1, 2, \ldots$, and $\gamma = 1/2$ the distribution of the square root of the $\chi^2$-distributed random variable with $n$ degrees of freedom.”

14. Appendix 1 item 25 p. 139 l.17: $\theta$ should be $\gamma$

15. Appendix 1 item 25 p. 140 l.6,10:

$$\exp\left(\frac{|\gamma| x^2}{2}\right) \text{ should be } \exp\left(-\frac{|\gamma| x^2}{2}\right)$$

16. Appendix 1 item 25 p. 140 l.7,11:

$$\exp\left(\frac{|\gamma| y^2}{2}\right) \text{ should be } \exp\left(-\frac{|\gamma| y^2}{2}\right)$$

17. Appendix 1 item 25 p. 140 l.12: $\theta$ should be $\gamma$

18. Appendix 1 item 26 p. 141 l-2: $\theta$ should be $\gamma$

19. Appendix 1 item 26 p. 142 l.-8,-4,:

$$\exp\left(-\frac{|\gamma| x}{2}\right) \text{ should be } \exp\left(\frac{|\gamma| x}{2}\right)$$

20. Appendix 1 item 26 p. 142 l.-7,-3,:

$$\exp\left(-\frac{|\gamma| y}{2}\right) \text{ should be } \exp\left(\frac{|\gamma| y}{2}\right)$$

21. Appendix 1 item 26 p. 142 l.-2: $\theta$ should be $\gamma$

22. Appendix 1: added a section on CEV processes.

23. Part II: right hand sides of the following formulas should be divided by $|p - q|$:

a. 1.1.6.4, 1.1.6.8, 1.1.27.4, 1.1.27.8, 1.1.29.8,
b. 2.1.6.4, 2.1.6.8, 2.1.27.4, 2.1.27.8, 2.1.29.8,
c. 3.1.6.4, 3.1.6.8, 3.1.27.4, 3.1.27.8, 3.1.29.8,
d. 5.1.6.4, 5.1.6.8, 5.1.27.4, 5.1.27.8,
e. 9.1.6.4.

The right hand side of the inverse Laplace transform formula k. in Appendix 3 p. 649 should also be divided by $|p-q|$. For Formula 5.1.29.8 see below.

24. Part II, Formula 3.1.13.3 p. 344: $*sc_t(y)$ should be $*sc_t(x, y)$

25. Part II, Formula 3.1.13.4 p. 345: $*sl_{t-v}(y)$ should be $*sl_{t-v}(x, y)$

26. Part II, Formula 5.1.29.6, p. 461: R.H.S. should be:

$$\frac{\lambda e^{-\lambda(u+v)}}{x} B_x^{(29)}(u, v, y, z) \, du \, dv \, dy \, dz \quad \text{for} \quad B_x^{(29)}(u, v, y, z) \quad \text{see 1.1.29.6.}$$

27. Part II, Formula 5.1.29.8, p. 461: R.H.S. should be:

$$\frac{z \Pi_{[q \wedge p), (q \vee p)](v)}{x|p-q|} B_x^{(29)} \left( \frac{|qt-v|}{|p-q|}, \frac{|pt-v|}{|p-q|}, y, z \right) \, dv \, dy \, dz \quad \text{for} \quad B_x^{(29)}(u, v, y, z) \quad \text{see 1.1.29.6}$$

28. Part II, Formula 7.1.9.8 (1) p. 526 is replaced by the following formula for the density

$$P_0 \left( \int_0^t U_s^2 \, ds \in dy \Big| U_t = 0 \right) e^{-\theta t/2 + \theta y/4\sigma^2}$$

$$= \sqrt{\sigma} \sqrt{1-e^{-2\theta t}} \sum_{k=0}^{\infty} \frac{\Gamma(k+1/2)}{k!} e^{-(4k+1)^2 \sigma^2 \theta/8y} D_{3/2} \left( \frac{(4k+1)\sigma \sqrt{\theta}}{\sqrt{2y}} \right) dy$$

29. Part II, Formula 8.1.28.1 p. 578: divide RHS by $\theta$.

30. Appendix 2 p. 637: Expanded; in particular, a section on hypergeometric functions is included