

Let x be your guess about #euros in the jar. Assuming x has normal distribution with mean μ and variance σ^2 , the expected gain of your guess is approximately (neglecting discreteness of the true number) equals:

$$x \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right) \quad (1)$$

To maximize this we first calculate the derivative:

$$\frac{d}{dx} x \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right), \quad (2)$$

which equals

$$\frac{1}{2} \frac{\sqrt{2}}{\sqrt{\pi}\sigma^3} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right) (-x^2 + \mu x + \sigma^2). \quad (3)$$

Then, we set the expression equal to zero and solve for x , assuming $\sigma > 0$:

$$\frac{1}{2} \frac{\sqrt{2}}{\sqrt{\pi}\sigma^3} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right) (-x^2 + \mu x + \sigma^2) = 0, \quad (4)$$

which yields the two roots:

$$\left\{ \frac{1}{2}\mu - \frac{1}{2}\sqrt{4\sigma^2 + \mu^2}, \frac{1}{2}\mu + \frac{1}{2}\sqrt{4\sigma^2 + \mu^2} \right\}, \quad (5)$$

and since the answer cannot be negative, we end with the optimal answer:

$$\frac{1}{2}\mu + \frac{1}{2}\sqrt{4\sigma^2 + \mu^2}. \quad (6)$$

For instance, if $\mu = 500$ and the standard deviation $\sigma = 75$, we get:

$$x = \frac{1}{2}500 + \frac{1}{2}\sqrt{4 \cdot 75^2 + 500^2} \approx 511. \quad (7)$$

Thus we notice that the optimal answer is higher than the most probable value μ .