FIRM VALUATION WITH TAXES



Firm Valuation with Corporate Taxes

Assumptions:

- Only corporate taxes - individual tax rate is zero
- Capital markets are frictionless
- Individuals can borrow and lend at the risk-free rate
- There are no costs to bankruptcy

- Firms issue only two types of claims: risk-free debt & (risky) equity
- All firms are in the same risk class

$$CF_i = \lambda CF_j$$

- No other taxes than corporate taxes
- All cash flow streams are perpetuities
- Everybody has the same information
- No agency costs

The P/L statement

Let

NOI = Net Operating Income

t = tax rate

D = outstanding debt

r = interest rate

P = net profit

ρ = return requirement on equity

Assume first that D=0. Then P = NOI-t*NOI= NOI*(1-t)

The value of the firm's equity - under the going concern assumption - is equal to the value of the firm:

$E^U = V^U = P/\rho = NOI*(1-t)/\rho$

The P/L statement

Assume next that $D>0$.

Then P = NOI-rD-t*(NOI-rD)= (NOI-rD)*(1-t)

The firm now has to yield profits to cover both return on equity and interest on loans. The value of the firm's equity - under the going concern assumption – is

$E^{L} = NOI*(1-t)/\rho - rD(1-t)/r = E^{U} - D(1-t)$, from which

$E^{U} = V^{U} = E^{L} + D(1-t) = E^{L} + D - Dt.$

But $V^{L} = E^{L} + D$ by definition. Thus,

$V^{U} = V^{L} - Dt.$ Notice that if t=0, then

$V^{U} = V^{L}$ (the famous Modigliani-Miller theorem)

The Modigliani-Miller theorem implies that

"The market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate ρ appropriate to its risk class"

(Modigliani-Miller, American Economic Review, 1958 june)

Hamada's equation

The beta of a firm *i* – obtained from ordinary least squares regression – is: $\beta_i = \mathrm{cov}(r_i, r_M)/\mathrm{var}(r_M)$ (1)

From the discussion above, we know that the return on equity for a firm without debt – an unlevered firm – is
$r_U = \mathrm{NOI}^*(1-t)/E_U$ (2)
and for a firm having debt – a levered firm – is
$r_L = (\mathrm{NOI}-rD)^*(1-t)/E_L$. (3)

Inserting (2) and (3) in (1) gives the betas of the unlevered and levered firms:

Hamada's equation

$$\beta_U = \frac{\mathrm{cov}(NOI(1-t)/E_U, r_M)}{\mathrm{var}(r_M)} \Rightarrow E_U\beta_U = \frac{\mathrm{cov}(NOI(1-t), r_M)}{\mathrm{var}(r_M)}$$

and

$$\beta_L = \frac{\mathrm{cov}((NOI - rD)(1-t)/E_U, r_M)}{\mathrm{var}(r_M)} \Rightarrow$$

$$E_L\beta_L = \frac{\mathrm{cov}(NOI(1-t), r_M)}{\mathrm{var}(r_M)} - \frac{\mathrm{cov}(rD(1-t), r_M)}{\mathrm{var}(r_M)}$$

Hamada's equation

By elementary covariance rules, cov(x+y,z) = cov(xz)+cov(yz), giving the expression for β_L. Furthermore, since the interest rate is assumed to be nonstochastic, the latter term in the expression for β_L is zero. Therefore,

$$E_L\beta_L = \frac{\text{cov}(NOI(1 - t), r_M)}{\text{var}(r_M)} = E_U\beta_U \Rightarrow$$

$$\beta_L = \frac{E_U\beta_U}{E_L}$$

Hamada's equation

We showed above that $E^U = E^L + D - Dt$.

Inserting this in the expression for beta of the levered firm gives the well-known Hamada's equation [1972], combining the Modigliani-Miller theorem with the Capital Asset Pricing Model (CAPM):

$$\beta_L = \frac{E_U \beta_U}{E_L} = \frac{(E_L + D - Dt)\beta_U}{E_L} = (1 + \frac{D}{E_L}(1-t))\beta_U$$

- Assume that the firm's debt (*D*) equals the market value of the debt (*B*).
- When the firm makes an investment *I*, its value will change according to (C-W, p. 445)

$$\frac{\Delta V^L}{\Delta I} = \frac{(1-t_c) * \Delta E(NOI)}{\rho * \Delta I} + t_c \frac{\Delta B}{\Delta I}$$

- The above investment will affect the value of the levered firm:

$$\Delta V^{L} = \Delta S^{0} + \Delta S^{n} + \Delta B^{0} + \Delta B^{n}$$

Note that Equity = old + $\Delta S^{0} + \Delta S^{n}$

- Because the project has the same risk as those already outstanding, the value of the outstanding debt stays the same $(\Delta B^{0} = 0)$.

- Because the new project is financed with new debt, equity or both

$$\Delta I = \Delta S^{n} + \Delta B^{n}$$

- Inserting ΔI into the above formula,

$$\frac{\Delta V^{L}}{\Delta I} = \frac{\Delta S^{0}}{\Delta I} + \frac{\Delta S^{n} + \Delta B^{n}}{\Delta I}$$

$$\rightarrow \quad \frac{\Delta V^{L}}{\Delta I} = \frac{\Delta S^{0}}{\Delta I} + 1$$

- This means that the project has to increase the shareholders' wealth, so that

$$\frac{\Delta S^0}{\Delta I} = \frac{\Delta V^L}{\Delta I} - 1 > 0$$

$$\frac{\Delta V^L}{\Delta I} > 1 \qquad \text{and} \qquad \frac{\Delta S^0}{\Delta I} > 0$$

The Weighted Average Cost of Capital
(Copeland-Weston, 1988, p. 444)

- IAS 36.55: the discount rate should be the pre-tax rate that reflects current market assessments of the time value of money and the risks specific to the asset
- IAS 36.57: If market-determined discount rates are unavailable, one should recognize the entity's:
  - weighted average cost of capital
  - the incremental borrowing rate
  - other market borrowing rates

The Weighted Average Cost of Capital
(Copeland-Weston, 1988, p. 444)

- Recall the formula

$$\frac{\Delta V^{L}}{\Delta I} = \frac{(1-t_c)*\Delta E(NOI)}{\rho*\Delta I} + t_c \frac{\Delta B^{n}}{\Delta I} > 1$$

as shown it should be greater than 1, so

$$\frac{(1-t_c)\Delta E(NOI)}{\Delta I} > \rho(1-t_c \frac{\Delta B}{\Delta I})$$

- This results in what is called “the Weighted Average Cost of Capital”, WACC,(C-W, p.446).

$$WACC = \rho\left[1 - t_c \frac{\Delta B}{\Delta I}\right]$$

- If there are no taxes the cost of capital is independent of capital structure.

What does $\frac{\Delta B}{\Delta I}$ mean ?

- "If $\frac{B^*}{V^*}$ denotes the firm's long run target debt ratio ... then the firm can assume, that for any particular investment $\frac{dB}{dI} = \frac{B^*}{V^*}$ " (C-W, p. 446).

An alternative definition of the weighted average cost of capital

- Definition by Haley and Shall [1973]
- Target leverage ratio

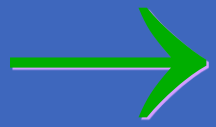
$$WACC = \rho\left[1 - t_c \frac{\Delta B}{\Delta V}\right]$$

Reproduction value

Reproduction value = PV of the stream of goods and services expected from the project.

How to calculate the cost of the two components in WACC (debt & equity)

- Assumptions:
- The cost of debt = k_b
- The cost of equity capital is the return on $\Delta S^0 + \Delta S^n$



$$\frac{\Delta NI}{\Delta S^0 + \Delta S^n}$$

This can be written as (C-W, p. 449):

$$\frac{\Delta NI}{\Delta S^0 + \Delta S^n} = \rho + (1 - t_c)(\rho - k_b)\frac{\Delta B}{\Delta S^0 + \Delta S^n}$$

Since the total change in equity is

$$\Delta S = \Delta S^0 + \Delta S^n$$

, the cost of equity $k_s = \frac{\Delta NI}{\Delta S}$ can be written as

$$k_s = \rho + (1 - t_c)(\rho - k_b)\frac{\Delta B}{\Delta S}$$

- If the firm has no debt in its capital structure, then $k_s = \rho$

It can be shown that (C-W, p. 451) WACC can be written as:

$$WACC = (1 - t_c)k_b \frac{B}{B+S} + k_s \frac{S}{B+S}$$

tax shield

cost of debt

Percentage of debt in the capital structure

cost of equity

Percentage of equity in the capital structure

- This formula is the same as the Modigliani-Miller definition

$$WACC = \rho\left[1 - t_c \frac{B}{B+S}\right]$$

→ The M-M and the traditional definition are identical !

Combining cost of equity equations :

$$R_f + [E(R_m) - R_f]\beta_L = \rho + (\rho - k_b)(1 - \tau_c)\frac{B}{S}$$

Cost of equity (CAPM) Cost of equity (MM)

Then substituting the CAPM definition of unlevered ρ, rearranging and canceling terms...

$$[E(R_m) - R_f]\beta_L = [E(R_m) - R_f]\left[1 + (1 - \tau_c)\frac{B}{S}\right]\beta_U$$

- If we can observe β_L by using observed rates of return on equity in the stock market, then we can also estimate β_U of the firms operating cash flows.

- In absence of bankruptcy costs, it doesn't matter if the debt is risky or not, the V^L is still the same ($V^L = V^U + t_c B$) .

C-W, 1988, p. 457.

Valuing the Equity of a Firm with the B & S Option Pricing Model (OPM)

- The equity of a firm can be seen as a call option.

➥ It is possible to apply the Black & Scholes OPM on the valuation problem.

$$S = VN(d_1) - e^{-r_f T} DN(d_2)$$

$$d_1 = \frac{\ln(V/B) + r_f T}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

C-W, p. 467.

,where

S = The market value of equity

V = The market value of the firm's assets

r_f = The risk-free rate

T = The time to maturity

B = The face value of debt (book value)

N(.) = The cumulative normal probability of the unit normal variate, d_1

σ = The standard deviation of the returns on the firm's assets

Measuring risk (beta) with the OPM

β_S = Systematic risk of equity

β_V = Systematic risk of the firm's assets

V = Market value of the firm's assets

S = Market value of equity

(according to Black & Scholes)

$$\beta_S = N(d_1)\frac{V}{S}\beta_V$$

- By substituting the above formula for S, we get

$$\beta_s = \frac{VN(d_1)}{VN(d_1) \quad Be^{r_f T} N(d_2)} \beta_v$$

$$= \frac{1}{1 \quad (B/V)e^{r_f T}[N(d_2)/N(d_1)]} \beta_v$$

The impact of parameters on β_S

- β on equity increases monotonically with leverage.

Increasing parameter	β_S
Market value (V)	↙
Debt (B)	↗
Risk-free rate (r_f)	↙
Variance of the firm's assets' value (σ)	↙
Maturity of debt (T)	↙

C-W, p. 467.

The cost of debt funding (C-W, p. 469).

- According to the OPM k_B is

$$k_B = r_f + (\rho - r_f) * N(-d_1) * \frac{V}{B}$$

Risk premium

- The OPM, CAPM & M-M are all consistent, when there exist no bankruptcy costs!

C-W, p. 471.

Literature

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